Endogenous Uncertainty and Credit Crunches[*]

Ludwig Straub — Harvard

Robert Ulbricht — Boston College

January 16, 2023

Abstract

We develop a theory of endogenous uncertainty in which the ability of investors to learn about rm-level fundamentals is impaired during nancial crises. At the same time, higher uncertainty reinforces nancial distress. Through this two-way feedback loop, a temporary nancial shock can cause a persistent reduction in risky lending, output, and employment that coincides with increased uncertainty, default rates, credit spreads and disagreement among forecasters. We embed our mechanism into standard real business cycle and New-Keynesian models and show how it generates endogenous and internally persistent processes for the e ciency and labor wedges.

Keywords: Endogenous uncertainty, nancial crises, internal persistence.

JEL Classi cation: D83, E32, E44, G01.

[*] This paper supersedes an earlier working paper circulated in 2011 under the titl.97@2011 me,crd [(*)]TJ/F87 9.9691

1 Introduction

Financial crises often entail deep and long-lasting recessions (Reinhart and Rogoff, 2009; Hall, 2014; Ball, 2014). A common view gives a central role to uncertainty, both as an amplifier of financial distress and a source of slow recovery.[1] This paper explores this idea, developing a theory that formalizes the interaction between financial constraints and uncertainty.

Our theory provides a narrative of how a temporary shock emanates from the financial sector, is reinforced by endogenously rising uncertainty, and ultimately develops into a long-the persistence of the output response in our model is much greater than that, with a half-life of 16 quarters. The discrepancy is caused entirely by the interaction between endogenous uncertainty and financial frictions: when shutting down the former, the half life of the output response falls to 4 quarters, mirroring the half-life of the exogenous financial shock.

For illustrative purposes, our baseline model is stylized and does not feature capital. Nevertheless, as we demonstrate in three extensions, it is straightforward to incorporate our mechanism into richer environments. First, we explore a variant of our model, in which a fraction of firms does not rely on external funds to finance their projects. While the presence of such firms scales down the overall impact of financial shocks, we find that it changes little about their propagation through endogenous uncertainty and does not reduce the internal persistence.

Second, we extend our baseline model to include investment and capital. Interestingly, we show that our model — with its firm-level heterogeneity and two-way interaction between lending and beliefs about firm potential — is observationally equivalent to a standard real business cycle (RBC) model with endogenous processes for the economy's "efficiency wedge" and "resource wedge", in the spirit of Chari, Kehoe and McGrattan (2007). These wedges arising from our mechanism are different from the ones in existing models based on financial frictions such as Buera and Moll (2015) in their internal persistence after a financial shock.

Third, we develop a New Keynesian version of our model, with nominal rigidities and hand-to-mouth households, following Galí, Lopez-Salido and Valles (2007) and Bilbiie (2008). We show that in this extension, as well, financial shocks lead to a protracted decline in output due to endogenous uncertainty. However, in contrast to our baseline model, the propagation now runs through the demand side, driven by a reduction in household income and consumer spending, manifesting itself as a persistent increase in the economy's labor wedge.

While the aggregate dynamics of the model are fully captured by endogenous wedges, our model also has implications at the firm level. In particular, as mentioned above, rising uncertainty helps explain a variety of financial market characteristics associated with financial crises: increased credit spreads, a rise in default rates, an increased cross-sectional dispersion of firm sales, and high levels of disagreement among forecasters about firm-level profitability.

To gauge the quantitative potential of our endogenous uncertainty mechanism, we estimate the RBC version of our model to historical data on U.S. business cycles, allowing for three typical business cycle shocks and the financial shock. We find that typical recessions, driven by the standard business cycle shocks, look similar with and without endogenous uncertainty. Recessions partly caused by financial shocks, however, are significantly more severe in the economy with endogenous uncertainty compared to an exogenous uncertainty counterfactual. In case of the Great Recession, we find that without endogenous uncertainty, the peak-totrough drop in output would have been about half of what it was, and output would have fully recovered by 2010.

All uncertainty in our model is about rm-level fundamentals, not aggregate fundamentals.

levels of uncertainty are particularly prevalent during financial crises.[4]

In our model, the emergence of uncertainty due to financial distress interacts with the propagation of uncertainty through the financial sector. In support of such a financial transmission channel, Gilchrist, Sim and Zakrajsek (2016) present evidence that uncertainty strongly affects investment *via* increasing credit spreads, but has virtually no impact on investment when controlling for credit spreads. The financial transmission of uncertainty relates our model to a recent literature around Christiano, Motto and Rostagno (2014), Arellano, Bai and Kehoe (2019), and Gilchrist, Sim and Zakrajsek (2016), which stresses the importance of uncertainty or risk shocks in the financial sector, but treats these shocks as exogenous.[5]

The predictions of our model are also broadly consistent with a recent empirical literature that measures the effects of tightening financial constraints. Giroud and Mueller (2017) show that establishments of firms that are more likely to be financially constrained were heavily affected by falling collateral values (house prices). In fact, they show that the entire correlation of employment loss and house prices is explained by these arguably financially constrained firms. Similar in spirit, Chodorow-Reich (2013) and Huber (2018) document that firms borrowing from less healthy lenders experience relatively steeper declines in employment during the financial crisis, consistent with the interpretation that these firms faced tighter financial constraints. Our model clarifies how an intense but relatively short-lived financial crisis can still translate into persistent financial constraints for firms, making it much harder

2 Baseline Model

We study our mechanism in a neoclassical economy with a representative household, a competitive final goods sector, and a continuum of monopolistically competitive intermediate-goods firms. The latter are partially funded by a competitive banking sector. Time is discrete with an infinite horizon and is indexed by t. To illustrate the mechanism, our baseline model abstracts from capital, nominal rigidity and non-credit based funding. We study the consequences of adding those features to our model in Sections 5 and 6.

2.1 Environment

Firms. A competitive final-good sector combines intermediate goods, $\{Y_{i,t}\}_{i\in[0,1]}$, to produce final output, Y_t, using the technology

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\xi-1}{\xi}}\,\mathrm{d}i\right)^{\frac{\xi}{\xi-1}};$$

where $\xi > 1$ is the elasticity of substitution between input varieties. Profit maximization yields the demand for input i with price $p_{i,t}$,

$$Y_{i,t} = Y_t p_{i,t}^{-\xi}; \tag{1}$$

Conditional on period-t productivities and given a real wage w_t, firms choose $p_{i,t}$ to maximize operating profits,

$$_{i,t} \equiv p_{i,t} Y_{i,t} - w_t L_{i,t} \tag{4}$$

subject to (1) and (2).

Which firm produces using which of the two productivity levels is determined by two interacting frictions: a financial friction and an informational friction. We explain them next, beginning with the financial friction.

Financial friction. Each period has two sub-periods, a morning and an afternoon.

In the morning, firms choose whether to operate the baseline technology, with productivity $\tilde{A}$, or the risky technology, with productivity $A_{i,t}$. Operating the baseline technology entails an upfront operating cost of $\tilde{} > 0$, whereas operating the risky technology entails a larger upfront cost of $> \tilde{} > 0$. Importantly, the technology choice is made subject to an information set I_t (detailed below), which does not contain the current realization of $A_{i,t}$. This is why the "risky technology" is indeed risky. Conditional on their technology choice, firms then approach banks to finance the upfront cost $_{i,t} \in \{ , \tilde{} \}$.

In the afternoon, firms produce, goods are sold, wages are being paid, loans are repaid, and the household consumes.

We assume that a liquidity constraint prevents firms from using their afternoon profits to pay for the upfront cost $_{i,t}$. Instead, firms borrow from a competitive banking sector in the morning, at an interest rate $r_{i,t}$, and repay their loans in the afternoon. When a firm is unable to do so due to its operating profits falling short of the repayment,

$$_{i,t} < (1 + r_{i,t}) \; _{i,t}, \tag{5}$$

it defaults on its loan. We assume that in case of default, banks need to pay a cost verifying the firms' default a la Townsend (1979), amounting to the firm's profits $_{i,t}$.[6] For simplicity, we assume that these costs are not resource costs and instead transfer from banks to households. If a firm defaults, it gets a bankruptcy flag that precludes it from obtaining risky loans, and thus precludes it from operating the risky technology. At the beginning of each period, bankruptcy flags are removed with an exogenous recovery probability $\in (0, 1]$.

The interest rate

solution to the zero pro t condition[7]

$$(1 + r_{i;t}) \ 1 - P_t \quad {}_{i;t} < (1 + r_{i;t}) \ {}_{i;t}$$

banks' surplus T_t^{banks}. Taken together, T_t can be written as

$$T_t = \int_0^1 (\ _{i;t} - \ _{i;t})\, \mathrm{d}i.$$

Information friction. We consider a simple information structure where all learning is public and there is no aggregate uncertainty; i.e., agents have complete information about the history of $\ _t$ and the *shape* of the cross-sectional distribution over $A_{i;t}$. The only source of uncertainty is a lack of information about the productivities of the risky technology of each *individual* firm. Specifically, each period, after the technology adoption choice and before firms set prices, all agents observe the realized risky productivities for all firms adopting the risky technology. By contrast, for firms adopting the baseline technology, current risky productivities are only observed with an exogenous probability $\ \in [0; 1)$, independently across firms, and remain otherwise unknown. Let B_t denote the set of firms that either adopt the risky technology in period t or for which $A_{i;t}$ is exogenously revealed. Then the information available to agents in the morning of date t is

$$I_t = \ _t \cup \{A_{i;t-1}\}_{i \in B_{t-1}} \cup I_{t-1}.$$

These assumptions imply that the common belief entertained about each firm's risky productivity is log-normal at all times, allowing us to track the public beliefs in terms of each firms' expected log-productivity and the corresponding uncertainty,

$$\ _{i;t} \equiv \mathrm{E}_t[\log A_{i;t} | I_t] \qquad \ _{i;t} \equiv \mathrm{Var}_t[\log A_{i;t} | I_t].$$

Timing and market clearing. The timing of events within each period can be summarized as follows:

- *Morning:* Bankruptcy flags are removed with probability $\ $; firms choose their technology; firms approach banks for funding and pay the operating cost $\ _{i;t}$.
- *Afternoon:* Risky productivities $A_{i;t}$ are revealed for all firms operating the risky technology and with probability $\ $ for all other firms; firms hire labor, produce, set prices, and repay loans; if firms are unable to repay, they default and get a bankruptcy flag; dividends and transfers are paid; the household consumes.

In equilibrium, the representative household chooses C_t, L_t and B_t to maximize utility (7), firms choose their technology and set prices to maximize profits, banks lend if their zero profit condition can be satisfied at the competitive default premium, and markets clear: labor markets satisfy $\int_0^1 L_{i;t}\,\mathrm{d}i = L_t$, goods markets satisfy

$$Y_t = C_t + \int_0^1 {}_{i;t}\,\mathrm{d}i; \tag{9}$$

and asset markets satisfy $B_t = 0$ at all times t.

Below, we work with a parameterization of the model in which firms using the baseline technology always make positive profits; and in which firms that can get a bank loan for the risky technology always find it optimal to do so.[9] The former assumption ensures that firms prefer operating the baseline technology to exiting; the latter assumption ensures that the financial friction has an impact on firm behavior.

Discussion. Two ingredients are at the core of our model. First, firms rely, at least in part, on external finance, and access to external finance hinges on the perceived quality and risk of their production potential. We model this by assuming that there is an upfront operating cost that needs to be financed through loans. In this environment, more pessimistic and/or uncertain beliefs by financial markets naturally reduce access to loans, because they translate into greater default risk, raising credit spreads.[10] In our baseline model all firms have ex-ante the same reliance on external finance. Ex-post, the ones that are perceived as more productive have no issues securing funding at costs close to the internal bank rate $_t$. In Section 5.3, we demonstrate that our mechanism is robust to also allowing for ex-ante heterogeneity in reliance on external funding. We do so by letting some firms fund the operating cost frictionlessly (e.g., due to equity, retained earnings or available safe collateral).

Second, a lack of funding leads to a lack of information about firms' potential productivity. In our model, firms that do not operate the risky technology generate less information about its productivity $A_{i;t}$. In reality, the risky technology captures a firm's potential, which is ex-ante uncertain. The longer a firm remains underfunded, unable to reach and test its potential, the less clear it becomes how profitable it actually is. Observe that $A_{i;t}$ need not correspond to productivity in reality. It could equally well capture firm-specific demand shifters; the two are isomorphic from a modeling perspective.

Finally, while we formalize the impact of being constrained in terms of firm productivity, one may equivalently think of it in terms of variations in factor utilization or differences in returns across a firm's projects. When we calibrate the model in Section 4.1, we will

[9] We can state the former assumption formally as $\bar{A}^{\ \xi}(\ -1)^{\xi\ 1}Y_t/w_t^{\xi\ 1} > (1+\ _t)\tilde{\ }$. The latter assumption is more complex as firms internalize how uncertainty affects future access to credit and profits. We verify that it holds numerically in our calibration.

[10] As explored in an earlier draft of this paper, a similar logic applies if firms are funded through equity and equity investors are not fully diversified (Straub and Ulbricht, 2018).

Proposition 1. *Define the* (risky) lending threshold *as*

$$_t \equiv \log \left(1 + {}_t\right) - \log \left(Y_t / w_t^{\xi - 1}\right) + \log \left({}^{\xi} (- 1)^{1-\xi}\right) :$$

Firm i obtains funding for the risky technology if and only if (i) it has no bankruptcy flag, and (ii) the belief $({}_{i;t}; {}_{i;t})$ satisfies

$$_{i;t} - V({}_{i;t}) \geq {}_t \tag{13}$$

where $V( )$ is defined as

$$V( ) \equiv \min_{x \in (0;1]} \left\{ \Phi^{-1}(x) \sqrt{} - \log x \right\} :$$

Banks are willing to fund all risky projects for which ${}_{i;t} - V({}_{i;t})$ exceeds a time-varying threshold ${}_t$, which we henceforth refer to as *(risky) lending threshold*. We have $V(0) = 0$ and $V'( ) > 0$ for that is not too large, capturing that default becomes more likely as uncertainty increases, which in turn increases default premia and reduces the willingness of banks to lend. Only in the pathological case where default is more likely than repayment, V may decrease in . Henceforth, we assume that ${}_\epsilon$ is low enough so that V increases for $\leq {}^2_\epsilon/(1 - {}^2)$, which is easily satisfied numerically for reasonable unconditional variances of log revenue productivity documented in the data.[11]

Belief dynamics. The cross-sectional distribution of beliefs $({}_{i;t}; {}_{i;t})$ about productivities $A_{i;t}$ is a crucial state variable in our economy. From (3) we can derive the law of motion of beliefs about each firm i as

$$_{i;t+1} = \begin{cases} \log A_{i;t} + (1 -) \log A & \text{if } i \in B_t \\ {}_{i;t} + (1 -) \log A & \text{if } i \notin B_t \end{cases} \tag{14}$$

$$_{i;t+1} = \begin{cases} {}^2_\epsilon & \text{if } i \in B_t \\ {}^2 {}_{i;t} + {}^2_\epsilon & \text{if } i \notin B_t . \end{cases} \tag{15}$$

General equilibrium and steady state. Each firm i has an idiosyncratic state that is given by $S_{i,t} \equiv (A_{i,t}, \ {}_{i,t}, \ {}_{i,t}, d_{i,t})$ where $d_{i,t} \in \{0, 1\}$ is firm i's bankruptcy flag. In any given period, firm i's output and labor demand, $Y_{i,t}$ and $L_{i,t}$, are functions of its state $S_{i,t}$ as well as of the aggregates $(\ {}_t, w_t, Y_t)$,

$$Y_{i,t} = A_{i,t}^{\xi/(\xi-1)} \left(\frac{\quad}{\ -1}\right)^{-\xi} \frac{Y_t}{w_t^{\xi}} \qquad \text{and} \qquad L_{i,t} = A_{i,t} \left(\frac{\quad}{\ -1}\right)^{-\xi} \frac{Y_t}{w_t^{\xi}},$$

where $A_{i,t}$ is firm i's technology, determined by Proposition 1. Aggregating across firms, we find that

$$w_t = (1 - \ ^{-1}) A_t \qquad \text{and} \qquad Y_t = A_t L_t \tag{16}$$

where

$$A_t \equiv \left(\int_0^1 A_{i,t} \, \mathrm{d}i \right)^{\frac{1}{\xi \ 1}} \tag{17}$$

corresponds to the efficiency wedge in the economy, in the spirit of Chari, Kehoe and McGrattan (2007), and $1 - \ ^{-1}$ stems from the monopoly distortion induced by monopolistic competition. Together with the first order condition for household labor supply, $w_t = \ L_t^{1/\zeta} C_t$, we find

$$(1 - \ ^{-1}) A_t = \ L_t^{1/\zeta} \left(A_t L_t - \int_0^1 \ {}_{i,t} \, \mathrm{d}i \right). \tag{18}$$

Conditional on firms' technology choices, this equation admits a unique positive solution for L_t. The solution always satisfies $A_t L_t > \int_0^1 \ \mathrm{d}i$. Thus, output Y_t is uniquely determined given A_t.

3 Endogenous Uncertainty and Lending

We are now ready to study the interaction between credit and learning that is at the core

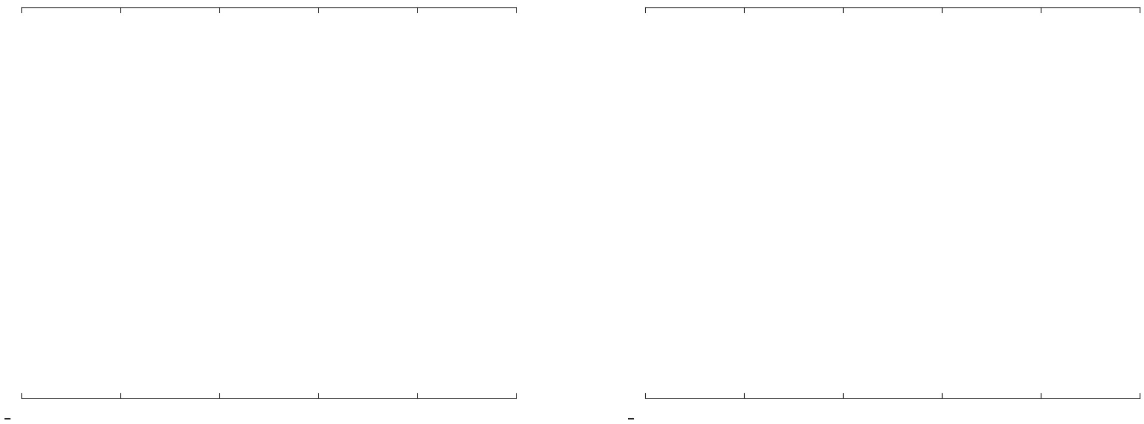


Figure 1: Phase diagram for rm-level beliefs in the absence of shocks

Note. Thin gray lines depict ($\mu - V(\ ) = \nu$)-contours; Z-shaped blue lines are the constant- $_{i;t}$ locus; vertical red lines are the constant-$\mu_{i;t}$ locus. Arrowheads represent one period in time along the plotted trajectories. Parameterization as in Section 4.1; c.f. Footnote 12. *Left:* Case with a unique steady state ($\nu < \bar{\nu}$). *Right:* Case with multiple steady states ($\underline{\nu} < \nu < \bar{\nu}$).

reduce uncertainty.

The \Z" shaped pattern visible in Figure 1 captures the self-reinforcing nature of endogenous uncertainty in our model. When uncertainty is high today, a rm is less likely to receive funding for the risky technology, which further increases uncertainty going forward. When is neither too low nor too high, this e ect can be su ciently strong to generate two steady states in the phase diagram. As our next proposition shows, and Figure 1 illustrates, this induced along the path. Along the rightmost trajectory, the firm is initially unconstrained and beliefs immediately adjust to the unique steady state. By contrast, along the two trajectories starting to the left of the gray contour line, the firm is initially denied risky funding so that learning breaks down. Accordingly, mean beliefs $_{i;t}$ only slowly converge to the unconditional prior, whereas uncertainty accumulates to higher and higher levels as information about past levels of $A_{i;t}$ becomes less and less useful for predicting current productivity. This, in turn, reinforces tight credit constraints. Hence, even though the steady state is unique, a firm can find itself lacking full access to credit for a significant period of time, unable to invest in their risky technology.

More generally, the duration without access to risky funding is governed by a "race" between the mean-reversion in $_{i;t}$ and rising uncertainty. Consider a marginally constrained firm with $_{i;t}$ just below $V(\,^2_\epsilon) + \,$. Stepping forward in time by one period, it will be constrained at $t+1$ if and only if

$$V(\,^2_\epsilon) - V((1 + \,^2)\,^2_\epsilon) < (1 - \,)(\, - \log A).$$

Hence, the marginally constrained firm will lose access to credit for multiple periods if either aggregate credit conditions are sufficiently bad (

Figure 2: Dynamic response to a financial shock at $t = 1$ and a subsequent recovery at $t = 2$

Note. Arrowheads represent one period in time along the plotted trajectory. Parameterization as in Section 4.1, with $\nu_0 = \nu_{2+s}$, $s \geq 0$, set to the value of ν at the aggregate steady state, and $\nu_1 = \nu^{ss} + 0.1$.

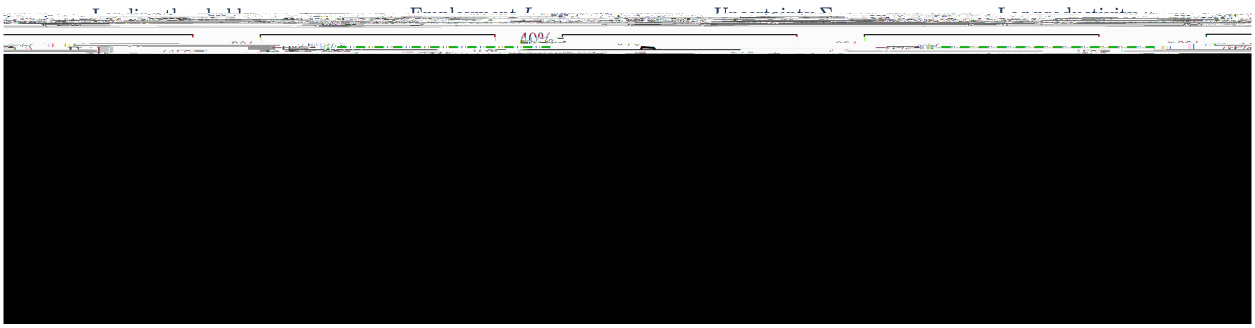


Figure 3: Impact of temporary financial shock on firm dynamics

Note. Black solid line: Effect of one time disruption in credit, $\nu_0 > \nu$, in period $t = 0$ on the average evolution of a firm close to the funding threshold, $\rho \log A = \nu + V\left(\sigma^2_\epsilon\right)$. Red dashed line: Same evolution, but fixing uncertainty exogenously at $_{i;t} = \sigma^2_\epsilon$. Parameterization as in Section 4.1.

solid gray line), even a reversal of $_t$ to does not end the feedback loop, generating internal persistence of the shock.[13]

Figure 3 repeats the experiment in our model with all firm-level shocks active, showing how the average evolution across different sample paths is affected by a one-period long disruption in credit. To isolate the contribution of the endogenous-uncertainty channel, we contrast the model's response (solid black lines) with a counterfactual response, in which the firm suffers the same exogenous financial shock but uncertainty is fixed at its lower bound, $= {}^2_\epsilon$ (dashed red lines). We call this the *exogenous uncertainty* model as a contrast with

[13]Here we initialized the firm close enough to the constraint so that uncertainty surpasses the original $(- V( ) =)$-contour line after one period. In general, an exogenous disruption in credit lasting for $T - 1$ periods cause internal persistence beyond the exogenous shock if $^T \log A < {}_0 + V\left(\frac{1 \ \rho^{2T}}{1 \ \rho^2} \ {}^2_\epsilon\right)$.

our *endogenous uncertainty* model. The exogenous uncertainty model will serve as a useful benchmark for the remainder of this paper.

In both cases, output initially drops due to the switch in technologies for the duration of the financial shock. The difference between our model and the exogenous-uncertainty counterfactual emerges at $t = 1$. Whereas output recovers in the counterfactual once access to credit is restored, the firm continues to be denied funding in the presence of endogenously increased uncertainty. The disruption in credit continues until either $_{i;t}$ crosses the $(\ - V(\ ) = \ )$-contour in Figure 2 or the potential productivity $A_{i;t}$ is exogenously revealed (with probability). In both cases, uncertainty drops to $^2_\epsilon$ and the firm switches back to the risky technology.

The dynamics shown in Figure 3 are reminiscent of the evidence in Huber (2018), who shows that a quasi-exogenous temporary financial shock can have a long-lasting effect on firm performance. In particular, Huber (2018) shows that the gap in employment between firms that were exposed to the shock and firms that were not remains elevated for two years after the shock.

3.3 Informational Externalities

We conclude this section with a brief discussion of efficiency. Our specification of credit constraints implies two sources of inefficiency. First, credit access is *statically inefficient* due to the presence of default costs, which give rise to the usual static wedge between supply and demand for credit.[14] Second, the combination of endogenous learning and external funding introduces a novel *dynamic inefficiency* that arises because atomistic banks do not internalize the option value of learning about a firm's risky technology. In our setup, this is because firms and banks cannot write contracts that are contingent on productivity realizations in future periods. This leads banks to lend too little.

The two inefficiencies suggest welfare gains from subsidizing bank lending. Interestingly, by mitigating the dynamic inefficiency, subsidized bank lending generates new information-9ne-

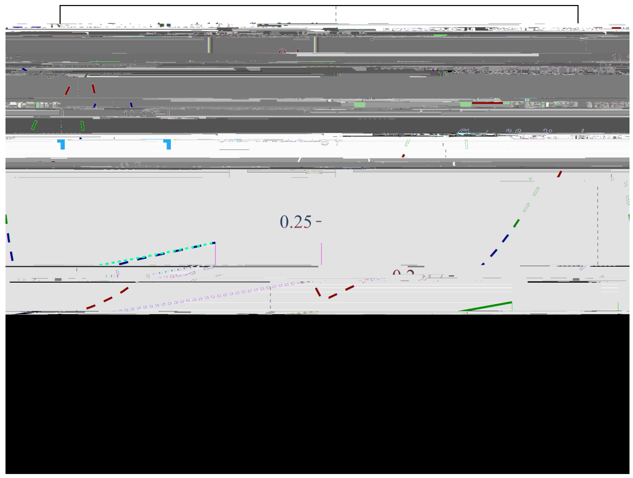


Figure 4:

private loan size exceeds the xed cost precisely to the right of the lending threshold (vertical dotted line), where $_{i;t} > \; + V$

Table 1: Calibrated parameters

Parameter					A	$\bar{A}=A$		ϵ			
Endog. uncertainty	0.99	2.000	5.000	0.913	1.041	0.563	0.944	0.073	0.139	0.350	0.117
Exog. uncertainty	0.99	2.000	5.000	0.907	0.983	0.563	0.944	0.073	0.121	0.350	1.000

our choice of using two proxies for the fraction of rms that lack su cient funding. First,

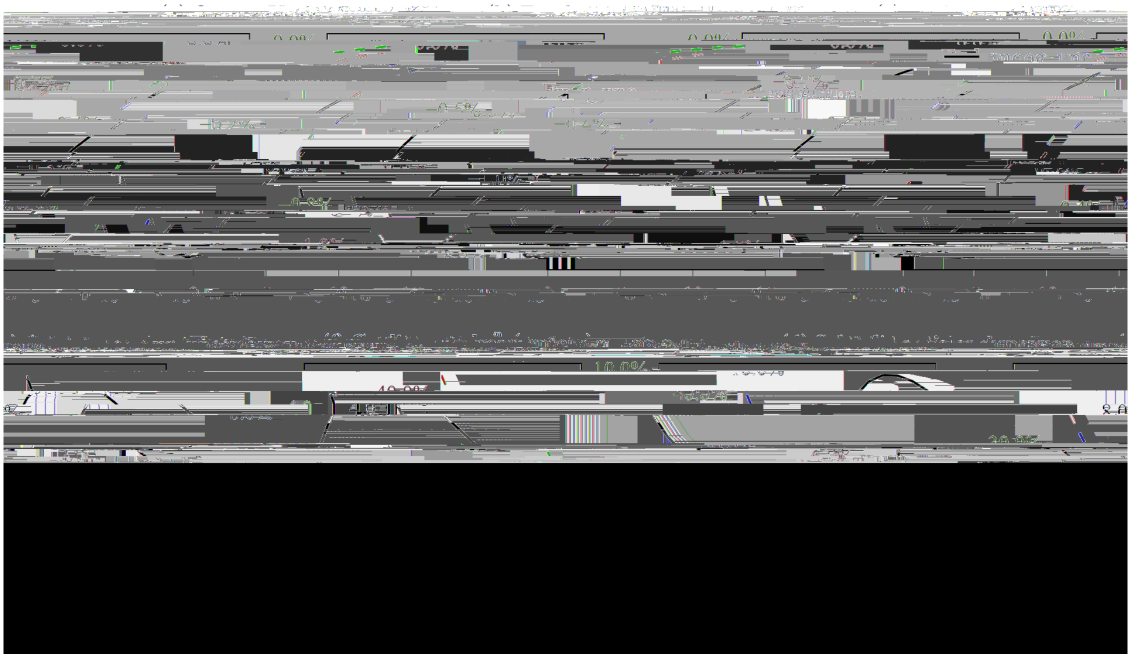


Figure 5: General equilibrium response to an AR(1) nancial shock

Note.

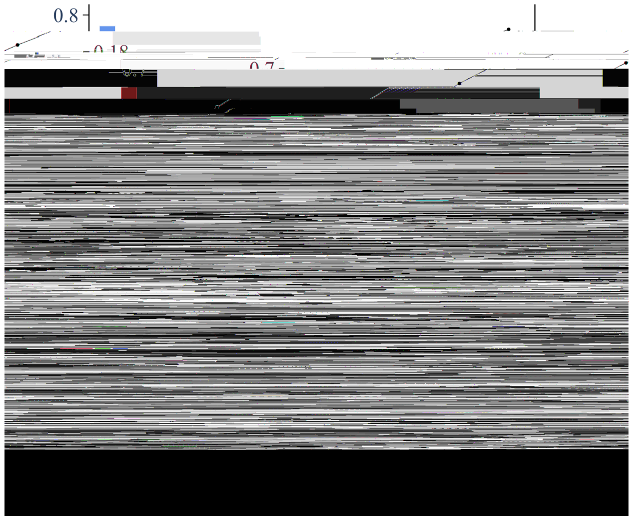


Figure 6: The distribution of uncertainty 5 periods into the impulse response and at the steady state

Note. Solid black line: level of uncertainty corresponding to *s*

Credit spreads, default rates, and dispersion. Rising uncertainty also helps explain a

In Appendix A, we use these forecasters' beliefs to compare the model's predictions with micro data from a survey of professional forecasters. From (20), the degree of "disagreement" among forecasters is given by

$$\mathrm{sd}_j[\tilde{\ }_{ij;t}] = \frac{{}^{-1}}{{}_{i;t}^{-1} + {}^{-2}}. \tag{21}$$

Thus, according to our model, there should be a tight empirical link between disagreement and the degree to which a firm is financially constrained. As shown in Appendix A, this is indeed the case.

5 Extensions

We next present three extensions that demonstrate how our mechanism operates (i) in the presence of investment and capital, (ii) in a New Keynesian version of our model, and (iii) when some firms do not rely on external funds to finance their projects.

5.1 Introducing Capital

Our first extension introduces capital to the baseline model in Section 2 and compares it to a standard real business cycle (RBC) model. To do so, we modify the production function of firm i to a Cobb-Douglas aggregate of capital and labor

$$Y_{i;t} = A_{i;t}^{\frac{1}{\xi-1}} K_{i;t}^{\alpha} L_{i;t}^{1-\alpha};$$

where capital $K_{i;t}$ is rented at the competitive rate $1 + r_t^K > 0$ from households. The representative household is now allowed to not only save in bonds B_t (which are still in zero net supply) but also in capital K_t. The date-t budget constraint now reads

$$C_t + B_{t+1} + K_{t+1} = w_t L_t + (1 + r_{t-1}) B_t + \quad 1 + r_t^K - \quad K_t + T_t.$$

As usual, capital K_t is determined one period in advance. Market clearing,

$$K_t = \int_0^1 K_{i;t} \, \mathrm{d}i;$$

determines the rental rate $1 + r_t^K$

lending threshold $_t$ is now given by

$$_t = \log\left(1 + {}_t\right) - \log\left(\frac{Y_t}{(1 + r_t^K)^{\alpha(\xi-1)} w_t^{(1-\alpha)(\xi-1)}}\right) + \log\, {}^{\xi}(- 1)^{1-\xi}\,.$$

We next show that the model with capital is equivalent to an RBC model, with an endogenous process for TFP corresponding to the efficiency wedge introduced in (17) and an endogenous process for a resource wedge as defined below.

Proposition 3. *Conditional on processes of the efficiency wedge $\{A_t\}$, defined in (17), and a resource wedge $\{G_t\}$, defined by*

$$G_t \equiv \int_0^1 {}_{i,t}\, \mathrm{d}i, \tag{22}$$

the equilibrium behavior of $\{C_t, K_t, L_t\}$ (and therefore also of other aggregates, such as Y_t, w_t, r_t^K) is described by a standard RBC model,

$$C_t^{-1} = \mathrm{E}_t \left[(1 - {}^{-1}) \; A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \right] C_{t+1}^{-1} \tag{23}$$

$$L_t^{1/\zeta} = (1 - {}^{-1})(1 -)\, C_t^{-1} A_t K_t^{\alpha} L_t^{-\alpha} \tag{24}$$

Figure 7: General equilibrium response in the model with capital

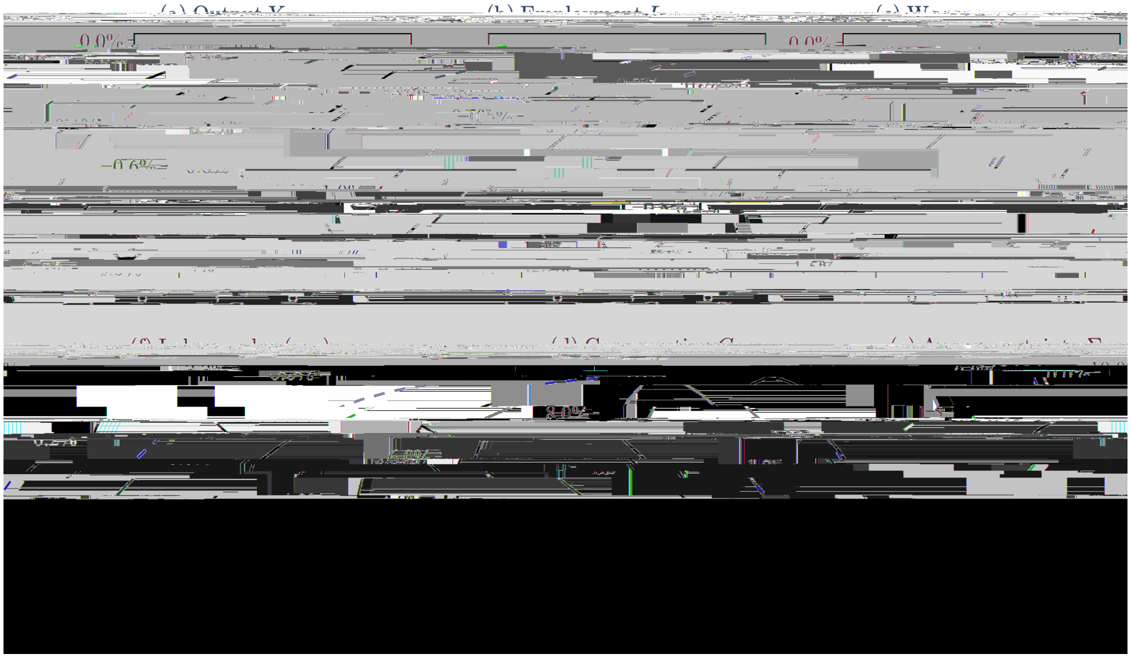


Figure 8: Response to nancial shock with nominal rigidities and hand-to-mouth agents

Note. All parameters as in Section 4.1. Share of hand-to-mouth agents of 50%.

an assumption we make here as well.[21] Our results here would qualitatively be very similar with an active Taylor rule, though quantitatively would depend on the exibility of nominal wages.

Aggregate consumption in this model is then characterized by

$$C_t = C_t^{\text{base}} + w_t L_t = 2$$

(C_t^{base})t =(1-17.2297701 Tf 0.859 -7.861 G 824(t)]TJ/F r Tf 17.4 2.955 Td [(61.)] 3)6]TJ/F111 7.9701 11.9552 Tf 5.426(3)]TJ 0 2 11.9553

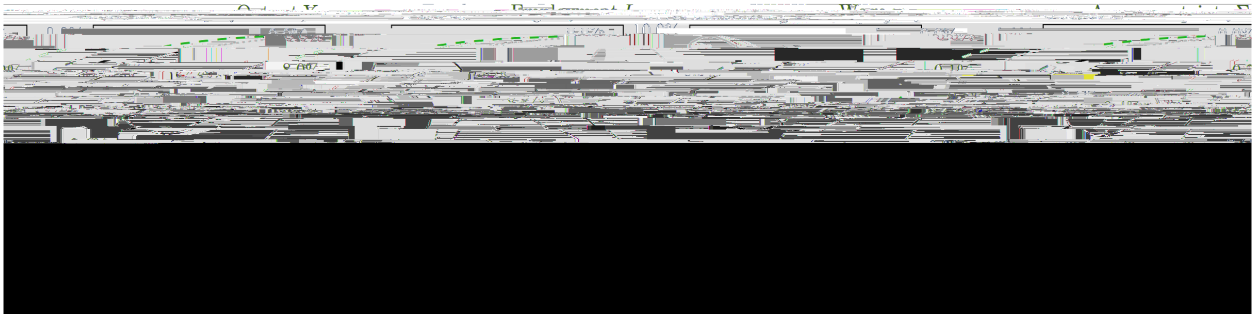


Figure 9: General equilibrium response in the model with both bank-financed and equity-financed firms

economy with nominal rigidities. By throttling new loans to firms, the financial shock directly reduces spending of firms and thus aggregate demand and aggregate income. This is then amplified via the Keynesian cross as hand-to-mouth households cut back on their spending in response to lower incomes. The financial shock and the associated decline in aggregate demand persistently tighten the lending threshold $_t$ in (19). Like before, in the endogenous uncertainty model this leads to a persistent decline in lending activity.

Interestingly, while we feed in a shock to the supply side of the economy, the shock ends up lowering aggregate demand sufficiently to cause a demand-driven recession, with a positive labor wedge, similar to the logic in Guerrieri et al. (2022) and the evidence in Huber (2018).

5.3 Introducing Equity-Financed Firms

So far, all firms equally relied on bank credit in order to fund their projects, exposing their ability to operate to the beliefs of the financial market. We now explore the case in which some firms are equity-financed and do not need bank credit to fund the fixed cost $_{i;t}$.[22] This allows them to always operate their preferred technology. To make it even starker, we assume away any information frictions for those firms as well. That is, equity-financed firms are able to observe $A_{i;t}$ at the end of each period, irrespective of the technology that was actually used in production. We explore the robustness of our mechanism to this extension by assuming that one half of all firms are equity-financed and thus never face any financial constraints.

Figure 9 shows the aggregate responses to a financial shock with the same magnitude as our baseline in Section 4. For comparison, we include the responses from the baseline model. Not surprisingly, the impact response is scaled down by the fraction of firms affected by the shock. Reassuringly, however, the responses are similarly persistent — if not more — compared

[22] Through other mechanisms, equity financing may also subject firms to the beliefs of the financial market, giving rise to a similar mechanism as the one in this paper. We explored this in a previous working paper version (Straub and Ulbricht, 2018).

with the baseline responses. Inasmuch as we do not have a strong prior about the magnitude of the exogenous shock, the two models hence behave very similarly in terms of measurable variables.

6 Quantitative Exploration

We next explore the quantitative relevance of endogenous uncertainty for businesses cycles. To do so, we build on the version of our model with capital (Section 5.1) and further extend it to allow for three additional standard shocks. We allow for shocks to total factor productivity (TFP) Z_t,

$$Y_{i;t} = Z_t \cdot A_{i;t}^{\frac{1}{\xi \; 1}} K_{i;t}^{\alpha} L_{i;t}^{1-\alpha};$$

shocks to the labor wedge ${}_t^L$, modifying the rst order condition for labor supply from (24) to

$$L_t^{1=\zeta} = \; 1 - \; {}_t^L \;\; (1 - \; {}^{-1})\,(1 - \;\;)\, C_t^{-1} A_t K_t^{\alpha} L_t^{-\alpha};$$

and shocks to the investment wedge ${}_t^I$, modifying the Euler equation from (23) to

$$1 + \; {}_{t-1}^{I}$$

Table 2: Priors and posteriors

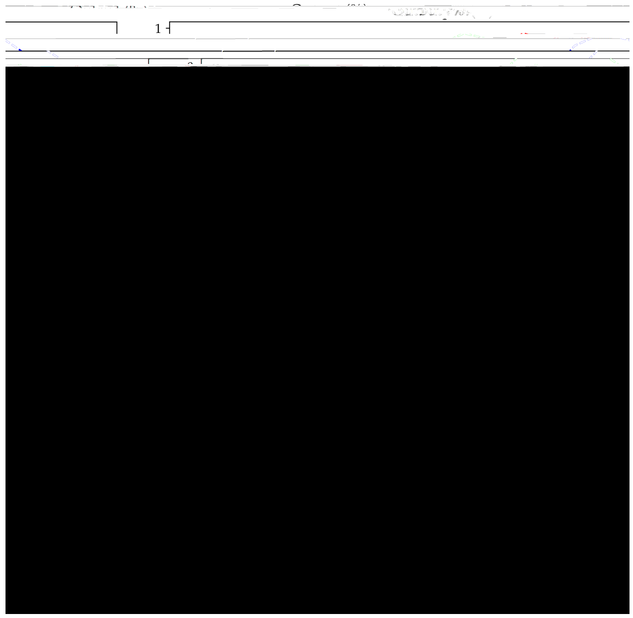


Figure 10: Contribution of endogenous uncertainty to the Great Financial Crisis

Note. This plot illustrates the role of endogenous uncertainty for the behavior of output and hours around the Great Financial Crisis. The black line is the data, which the endogenous uncertainty model matches exactly. The red line simulates the exogenous uncertainty model, subject to the same shock realizations as the endogenous uncertainty model. Both plots are normalized to 0 in 2008Q1.

with ω_t i.i.d., normal, with zero mean and variance σ^2_ω. To maximize the potential for aggregate uncertainty fluctuations, we assume that agents do not infer any information about Z_t from the cross-sectional distributions of prices, outputs, etc.

To characterize the uncertainty dynamics in this economy, define the aggregate input bundle in the economy as

$$X_t = \frac{Y_t}{\quad}$$

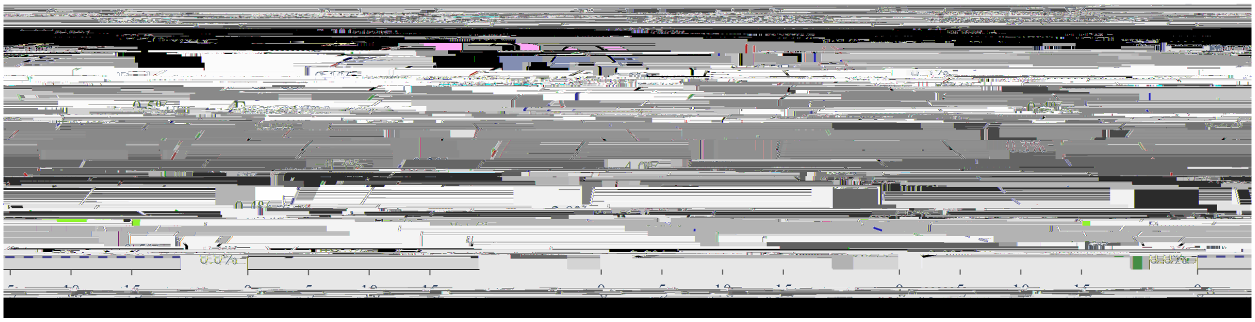


Figure 11: Endogenous uncertainty at the rm level vs. the aggregate level

$\mathrm{Var}[Z_t j I_t]$ converged to a constant. Now suppose the economy is hit by the same nancial shock as in Section 4, whereas aggregate productivity and the noise shock remain at their steady state values ($Z_{t+s} = Z$ and $!_{t+s} = 0$ for all $s \geq 0$). It then follows that agents' mean expectations remain unperturbed (i.e., $\mathrm{E}[Z_{t+s} j I_{t+s}] = Z$ for all s), and aggregate uncertainty evolves as follows

$$ {}^{Z}_{t} = \frac{{}^{2}_{Z}\ {}^{Z}_{t-1}}{1 + ({}_{!}=X_{t-1})^{-2}\ {}^{Z}_{t-1}} + {}^{2}_{Z}: \tag{28}$$

To maximize the potential impact of the aggregate uncertainty channel, we chose parameters ${}_{Z}$, ${}_{Z}$ and ${}_{!}$ so as to maximize the percentage increase in ${}^{Z}_{t}$ at the peak of the impulse response. Clearly, the response is maximized for ${}_{Z} = 1$. Moreover, because any proportionate scaling of ${}_{Z}$ and ${}_{!}$ also scales ${}^{Z}_{t}$ (and thus leaves the percentage response relative to steady state unchanged), it is su cient to set the relative standard deviation ${}_{!}= {}_{Z}$. We o

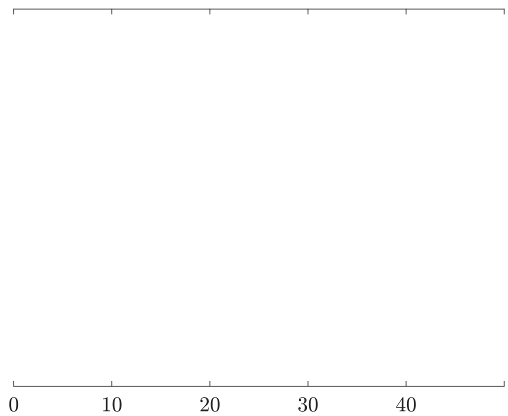


Figure 12: Peak increase in uncertainty by size of the financial shock in the aggregate uncertainty model

parameterization. It can be seen that for any magnitude of the shock, the peak increase in uncertainty is proportionately smaller than the corresponding loss in output. This is markedly different in our model with endogenous firm-level uncertainty. There, no matter how small the financial shock, it always results in some firms losing risky funding at the margin, starting the adverse credit–uncertainty spiral for those firms.

8 Concluding Remarks

References

Acharya, Sushant. 2013. \Dispersed Beliefs and Aggregate Demand Management." *Mimeo*.

Albagli, Elias. 2011. \Ampli cation of Uncertainty in Illiquid Markets." *Mimeo*.

Amador, Manuel, and Pierre-Olivier Weill. 2010. \Learning from Prices: Public Communication and Welfare." *Journal of Political Economy*, 118(5): 866{907.

Amador, Manuel, and Pierre-Olivier Weill. 2012. \Learning from Private and Public Observations of Others' Actions." *Journal of Economic Theory*, 147(3): 910{940.

Angeletos, George-Marios, and Jennifer La'O. 2010. \Noisy Business Cycles." *NBER Macroeconomics Annual*, 24.

Angeletos, George-Marios, and Jennifer La'O. 2013. \Sentiments." *Econometrica*, 81(2): 739{779.

Arellano, Cristina, Yan Bai, and Patrick J. Kehoe. 2019. \Financial Frictions and Fluctuations in Volatility." *Journal of Political Economy*, 127(5): 2049{2103.

Asriyan, Vladimir, Luc A. Laeven, and Alberto Martin. 2021. \Collateral Booms and Information Depletion." *Review of Economic Studies*, forthcoming.

Auclert, Adrien, Bence Bardoczy, Matthew Rognlie, and Ludwig Straub. 2021. \Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models." *Econometrica*, 89(5): 2375{2408.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub. 2018. \The Intertemporal Keynesian Cross." *NBER Working Paper No. 25020*.

Bachmann, Rüdiger, and Christian Bayer. 2013. \'Wait-and-See' Business Cycles?" *Journal of Monetary Economics*, 60(6): 704{719.

Ball, Laurence. 2014. \Long-Term Damage from the Great Recession in OECD Countries." *European Journal of Economics and Economic Policies: Intervention*, 11(2): 149{160.

Baxter, Marianne, and Robert G. King. 1999. \Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series." *Review of Economics and Statistics*, 81(4): 575{593.

Benhabib, Jess, Xuewen Liu, and Pengfei Wang. 2019. \Financial Markets, the Real Economy, and Self-Ful lling Uncertainties." *The Journal of Finance*, 74(3): 1503{1557.

Bilbiie, Florin O. 2008. \Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic." *Journal of Economic Theory*, 140(1): 162{196.

Blanchard, Olivier J. 2009. \(Nearly) Nothing to Fear but Fear Itself." *The Economist*, January 31: 84.

Bloom, Nicholas. 2009. \The Impact of Uncertainty Shocks." *Econometrica*, 77(3): 623{685.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry. 2018. \Really Uncertain Business Cycles." *Econometrica*, 86(3): 1031{1065.

Boppart, Timo, Per Krusell, and Kurt Mitman. 2018. \Exploiting MIT Shocks in Heterogeneous-Agent Economies: The Impulse Response as a Numerical Derivative." *Journal of Economic Dynamics and Control*, 89: 68{92.

Born, Benjamin, Sebastian Breuer, and Ste en Elstner. 2018. \Uncertainty and the Great Recession." *Oxford Bulletin of Economics and Statistics*, 80(5): 951{971.

Buera, Francisco J., and Benjamin Moll. 2015. \Aggregate Implications of a Credit Crunch." *American Economic Journal: Macroeconomics*, 7(3): 1{42.

Caldara, Dario, Christina Fuentes-Albero, Simon Gilchrist, and Egon Zakrajsek. 2016. \The Macroeconomic Impact of Financial and Uncertainty Shocks." *European Economic Review*, 88: 185{207.

Chahrour, Ryan, and Gaetano Gaballo. 2021. \Learning from House Prices: Ampli cation and Business Fluctuations." *Review of Economic Studies*, forthcoming.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. \Business Cycle Accounting." *Econometrica*, 75(3): 781{836.

Chodorow-Reich, Gabriel. 2013. \The Employment E ects of Credit Market Disruptions: Firm-level Evidence from the 2008-9 Financial Crisis." *The Quarterly Journal of Economics*, 129(1): 1{59.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. \Risk Shocks." *American Economic Review*

Fajgelbaum, Pablo, Edouard Schaal, and Mathieu Taschereau-Dumouchel. 2017. \Uncertainty Traps."

Endogenous Uncertainty and Credit Crunches
| Online Appendix |

Ludwig Straub Robert Ulbricht

A Evidence From Survey Data

At the core of our model is a two-way interaction between uncertainty and nancial constraints, causing both variables to co-move. In this appendix section, we explore the extent to which this co-movement can be seen empirically, both in the micro-data and at the aggregate.

A.1 Data

Our dataset is a yearly panel of public US rms.

Proxies for uncertainty. Our proxy for uncertainty is based on forecasts about earnings

Proxies for financial constraints. For the purpose of measuring financial constraints, we follow the corporate finance literature and combine various balance sheet data to proxy for firms' access to funds. Our main measure is the "KZ-index" developed by Kaplan and Zingales (1997) and Lamont, Polk and Saa-Requejo (2001). Specifically, the "kz-score" of firm i at date t is given by

$$\mathrm{kz}_{i,t} = -1.001909 \times \frac{\text{cash flow}_{i,t}}{k_{i,t-1}} + 0.2826389 \times Q_{i,t}$$

Table A.I: Financial constraints and uncertainty

	Data			
	(1)	(2)	(3)	(4)
Financially constrained	.081	.079	.079	.031
	(.012)	(.012)	(.012)	(.008)
Observations	47 342	47 342	47 335	46 141
Adj. R-sq.	.010	.023	.078	.709
Year × month FE	no	yes	yes	yes
Sector FE (4 digit)	no	no	yes	no
Firm FE	no	no	no	yes

Note. Standard errors clustered at the firm-level are in parenthesis.

A.2 Financial Constraints and Uncertainty

Cross-sectional evidence. To explore whether the predicted link between financial constraints and uncertainty is present in the data, we run a simple OLS regression of forecast-error dispersion $^{fce}_{i;t}$ on the KZ-based indicator. Table A.I reports the estimated coefficients, controlling for different combinations of fixed effects. The estimated effect is roughly constant over

Figure A.I: Average forecast error dispersion (a proxy for uncertainty) of constrained and unconstrained rms.

Note. This gure shows the average forecast error dispersion among nancially constrained and among nancially unconstrained rms. Financially constrained rms are those whose current Kaplan and Zingales (1997) index lies in the top 5% of the distribution. Financially unconstrained rms are all other rms.

Table A.II: Alternative proxies for nancial stress

	(1)	(2)	(3)	(4)
Panel a: Financial conditions measured by dividends				
E ect of constraint	.030	.026	.018	-.003
	(.002)	(.002)	(.003)	(.002)
Observations	58 737	58 737	58 735	57 215
Adj. R-sq.	0.009	0.022	0.072	0.700
Panel b: Financial conditions measured by leverage				
E ect of constraint	.016	.014	.015	.003
	(.005)	(.005)	(.004)	(.0072)

second an indicator for whether the debt to capital ratio (which is a monotone function of leverage) is in the top 5% in a given year (Panel b).

The results are qualitatively similar to the ones in Table A.I. Quantitatively, the magnitudes in Table A.II are somewhat smaller compared to those in Table A.I. This is not surprising given that one may think of the KZ indicator as a (more or less) optimized indicator which already includes dividend payouts and leverage in its composition; and thus dividends and leverage are both relatively more noisy measures of financial constraints and therefore subject to greater attenuation bias.

B Mathematical Appendix

B.1 Proof of Proposition 1

Firm i at date t obtains a loan operating the risky technology if there exists an interest rate $r_{i;t} \geq \ _t$ such that

$$\Phi\left(\frac{\ _{i;t} - \log\ (1 + r_{i;t}) \quad + \log\ Y_t/w_t^{\xi-1} \ - \log\ \ ^{\xi}(\ - 1)^{1-\xi}}{\sqrt{\ _{i;t}}}\right) = \frac{1 + \ _t}{1 + r_{i;t}} \qquad \text{(A.1)}$$

Define $x \equiv \frac{1+\lambda_t}{1+r_{it}} \in (0;1]$. Equation (A.1) is equivalent to there existing an $x \in (0;1]$ such that

$$\ _{i;t} - \log\ (1 + \ _t) \quad + \log\ Y_t/w_t^{\xi-1} \ - \log\ \ ^{\xi}(\ - 1)^{1-\xi} = \Phi^{-1}(x)\sqrt{\ _{i;t}} - \log x$$

Observe that only the right hand side of this equation depends on x, and that it approaches infinity as $x \to 1$. Thus, the condition for a firm to be financed can be written as

$$\ _{i;t} - \log\ (1 + \ _t) \quad + \log\ Y_t/w_t^{\xi-1} \ - \log\ \ ^{\xi}(\ - 1)^{1-\xi} \geq V(\ _{i;t})$$

with

$$V(\ _{i;t}) \equiv \min_{x \in (0;1]} \left\{\Phi^{-1}(x)\sqrt{\ _{i;t}} - \log x\right\}.$$

This proves Proposition 1.

B.2 Properties of $V(\ )$

We prove a few properties of $V(\ )$ as well, which are stated in the text.

- $V(0) = \min_{x \in (0;1]} \{-\log x\} = 0$.

- There is a unique minimizer in the definition of $V(\ )$. To see this, note that the FOC reads

$$\frac{1}{\phi(\Phi^{-1}(x))}^{\sqrt{\ }} = \frac{1}{x}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the pdf and cdf of the standard normal distribution. Defining $z \equiv \Phi^{-1}(x) \in \mathbb{R}$, this can be rewritten as

$$\Phi(z)^{\sqrt{\ }} = \phi(z) \tag{A.2}$$

We claim that this is satisfied for a unique $z \in \mathbb{R}$. To see why, consider the derivatives of both sides

  –

where $A_t = F_t\left(\frac{Y_t}{w_t^{\xi-1}}\right)$ is an increasing function, and by (16), $w_t = (1 - {}^{-1})A_t$. Combining these equations, we nd a system of two equations and two unknowns, A_t and Y_t,

$$\frac{-1}{}A_t = \left(\frac{Y_t}{A_t}\right)^{1/\zeta}\left(Y_t - \int_0^1 {}_{i,t}\,\mathrm{d}i\right) \tag{A.3}$$

$$A_t = F_t\left(\frac{Y_t}{((1 - {}^{-1})A_t)^{\xi-1}}\right) \tag{A.4}$$

First, we observe that there always exists a solution to this system of equations. The reason is that (A.4) implies an increasing relationship between A_t and Y_t, which remains positive and bounded for $Y_t \to$

or

$$\log A - V($$

Since $K_{i;t}$ is proportional to $A_{i;t}$, this implies that

$$K_{i;t} = \frac{A_{i;t}}{A_t^{\xi-1}} K_t \tag{A.6}$$

where A_t is defined in (17). Similarly,

$$L_{i;t} = \frac{A_{i;t}}{A_t^{\xi-1}} L_t. \tag{A.7}$$

Using (A.5), (A.6) and (A.7) aggregate output is then given by

$$Y_t = \left(\int_0^1 Y_{i;t}^{\frac{\xi-1}{\xi}} \, \mathrm{d}i \right)^{\frac{\xi}{\xi-1}} = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{A.8}$$

with

$$r_t^K = (1 - \;^{-1}) \frac{p_{i;t} Y_{i;t}}{K_{i;t}} = (1 - \;^{-1}) \frac{Y_t}{K_t} = (1 - \;^{-1}) A_t K_t^{\alpha-1} L_t^{1-\alpha} \tag{A.9}$$

and similarly,

$$w_t = (1 - \;)(1 - \;^{-1}) A_t K_t^{\alpha} L_t^{-\alpha}. \tag{A.10}$$

The Euler equation from households is standard, and given by

$$C_t^{-1} = \mathrm{E}_t \quad 1 + r_{t+1}^K - \quad C_{t+1}^{-1}$$

Substituting in (A.9) yields (23). The optimality condition for labor is standard and given by

$$L_t^{1/\zeta} = C_t^{-1} w_t.$$

Substituting in (A.10) gives (24). Finally, the resource constraint (25) follows from (A.8).

C Solving the Model

We solve all variants of our model in the seque375(Mo)-31(del)]TJ/F42 11.9552 Q Td [6e,0.98 0/5 rg 0 31(de

- aggregate output Y_t
- e ciency wedge A_t
- total operating costs G_t

We compute these objects by iterating over the distribution of rms in belief space, $g_t(\; ; \; ; d)$ where $d \; 2 \; f0; 1g$ is an indicator for whether a rm is in default or not. Each period goes through the following stages:

- We start with the previous end-of-period distribution $g_t^{(0)} \equiv g_{t-1}$.
- We move a random fraction of defaulted rms back into no-default,

$$g_t^{(1)}(\; ; \; ; 0) = g_t^{(0)}(\; ; \; ; 0) + \; g_t^{(0)}(\; ; \; ; 1)$$
$$g_t^{(1)}(\; ; \; ; 1) = g_t^{(0)}(\; ; \; ; 1) - \; g_t^{(0)}(\; ; \; ; 1)$$

- We label by $\;_k$ the uncertainty associated with not having received a signal for k periods,

$$_0 = 0$$
$$_{k+1} = \; ^2 \; ^2_k + \; ^2_\epsilon \qquad k \geq 0$$

- We evolve beliefs to be over $\log A_{i;t}$ instead of $\log A_{i;t-1}$,

$$g_t^{(2)}(\; ; \; _{k+1}; d) = (1 - \;) \int \quad \int$$

~W g(2) t ; 2 1))3-327(costs)]TJ/F47 11.f 11.- Tf857f 11.9.7921252.955 Tc

{

- Aggregating, we nd aggregate output from

$$Y_t^{1-1=\xi} = \int y_t^{\mathrm{risky}}(\;)^{1-1=\xi} g_t(\;;\;_0;0)d \;+\; y_t^{\mathrm{base}\;1-1=\xi} \cdot \; 1 - \int g_t(\;;\;_0;0)d \quad ;$$

the e ciency wedge from

$$A_t^{\xi-1} = \int e^{\mu} g_t(\;;\;_0;0)d \;+\; \bar{A} \cdot \; 1 - \int g_t(\;;\;_0;0)d \quad ;$$

and total operating cost G_t from

$$G_t = \int g_t(\;;\;_0;0)d \;+\; \tilde{} \;\; 1 - \int g_t(\;;\;_0;0)d$$

We compute the Jacobian of this block as in the \forward iteration" step in Auclert et al. (2021).

2. Value added block [simple block]: The value added block maps the aggregate sequences for output Y_t, real marginal input cost mc_t, capital K_t, the e ciency wedge A_t, aggregate TFP Z_t, and the investment wedge $^{I}_{t}$ into

   - labor demand $L_t^d = \left(\frac{Y_t}{Z_t \mathbf{A}_t K_t^{\alpha}{}_{1}}\right)^{\frac{1}{1\;\alpha}}$
   - real wage $w_t = (1 - \;) \frac{\mathrm{mc}_t}{Z_t \mathbf{A}_t} \frac{Y_t}{L_t^d}$
   - return on capital $R_t = \frac{\mathrm{mc}_t}{Z_t \mathbf{A}_t} \frac{Y_t}{K_t}{}_{t}$ 1253.5561.7TJ/F1031.7TJ/F1031.7TJ/F1031.7TJ/F1031.7TJ/F1038

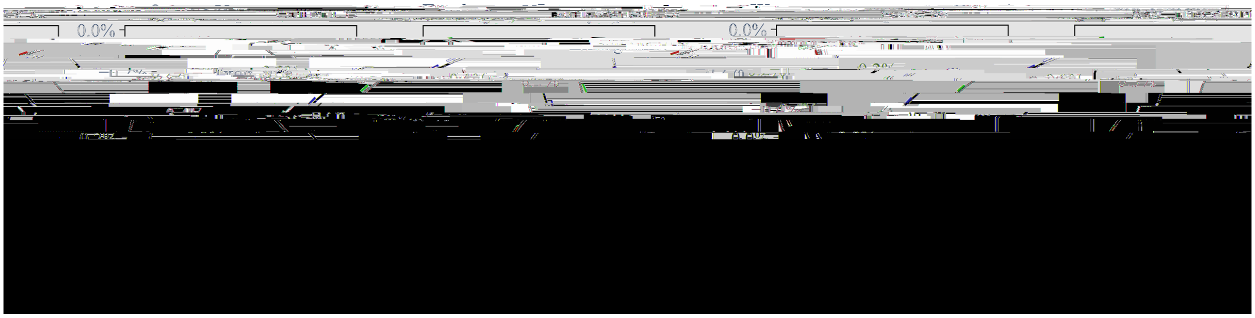


Figure A.II: Response to an aggregate productivity shock

- the Euler condition: $\text{euler}_t = 1 + r_{t+1} - {}^{-1}\frac{C_{t+1}}{C_t}$
- aggregate output condition: $\text{output mkt}_t \equiv Y_t - Y_t^d$

The three unknowns of this model are real marginal input cost mc_t, capital K_t, and aggregate demand Y_t^d. The three targets are labor mkt_t, euler_t, and output mkt_t. The four shocks are the nancial shock $_t$; TFP Z_t; the investment wedge $_t^I$; and the labor wedge $_t^L$.

D Additional Results

D.1 Aggregate Productivity Shocks

Our focus in this paper is on shocks to the nancial sector. One may wonder, however, whether our model with its nancial and information frictions also fundamentally alters the response to aggregate productivity shocks. To do so, suppose production is subject to a common, fully known, aggregate productivity shock Z_t,

$$Y_{i;t} = Z_t \cdot A_{i;t}^{\frac{1}{\xi - 1}} L_{i;t}.$$

Figure A.II shows that the endogenous and exogenous uncertainty models behave nearly identically in response to the aggregate productivity shock.[A3] This is because an aggregate productivity shock does not shift $_t$ nearly as much as the nancial shock, as the response of average uncertainty in Figure A.II shows.

D.2 Robustness to the Fraction of Financially Constrained Firms

In our calibration in Section 4.1, we worked with a parameterization that targeted a steady state share of 25% of constrained rms that do not have access to funding for the risky

A3

Table A.III: Parameters for robustness exercise

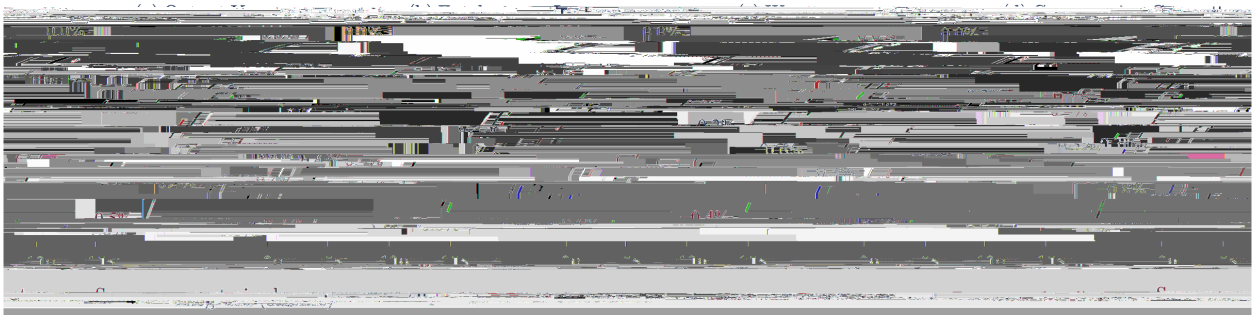


Figure A.IV: Comparing di erent ways of de ning the exogenous uncertainty benchmark

Note. Panels compare di erent ways of de ning the exogenous uncertainty benchmark.

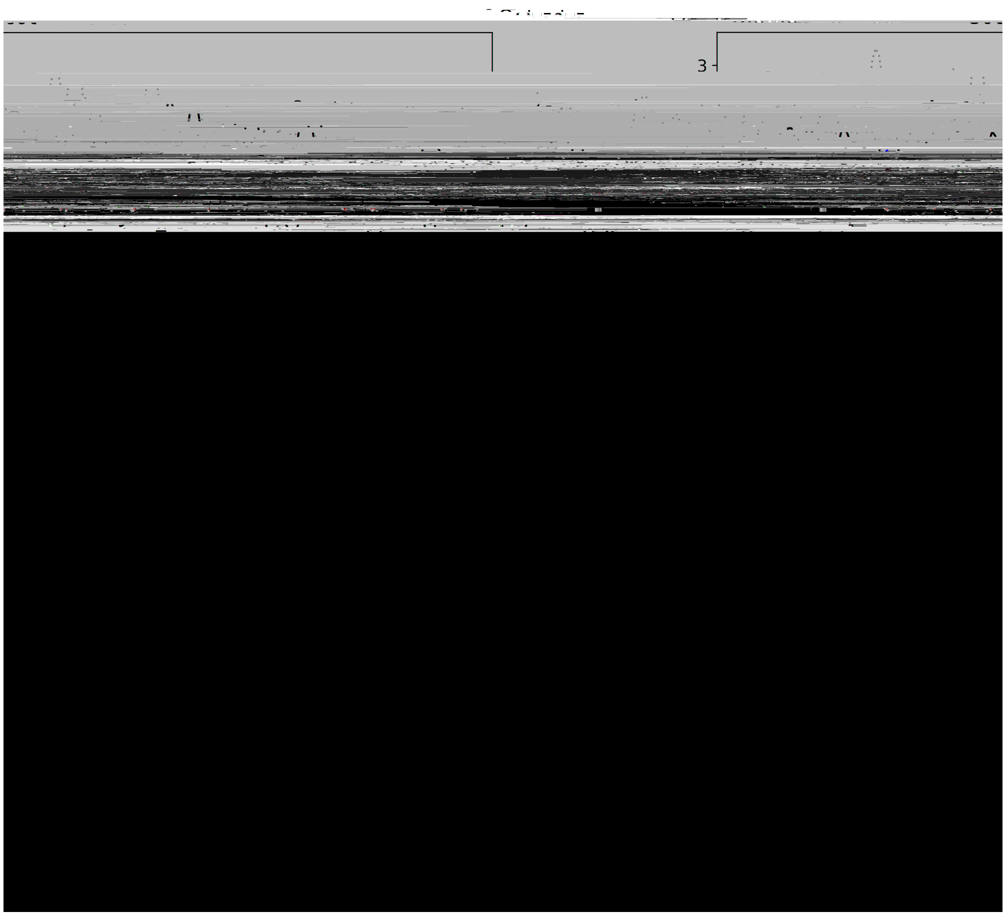
Table A.IV: Calibrated parameters of alternative models

Exog. uncertainty model					A	$\bar{A}/A$		ϵ			
Baseline	0.99	2.000	5.000	0.907	0.983	0.563	0.944	0.073	0.121	0.350	1.000
Same parameters	0.99	2.000	5.000	0.915	0.996	0.563	0.944	0.073	0.133	0.350	1.000
Same new constrained	0.99	2.000	5.000	0.899	0.978	0.563	0.944	0.073	0.112	0.350	1.000

D.4 Role of endogenous uncertainty for output and hours

Figure A.V illustrates the role of endogenous uncertainty for the historical paths of output and hours. To construct it, we start from the estimated endogenous uncertainty model, which is designed to match the data (dotted line). We plot the contribution of nancial shocks (black solid), where endogenous uncertainty matters most. The exogenous uncertainty benchmark (red dashed) is obtained by feeding the exact pattern of historical nancial shocks estimated for the endogenous uncertainty model into the model with exogenous uncertainty.

Figure A.V: Role of endogenous uncertainty for contribution of nancial shocks to the business cycle



Note. This plot shows the estimated historical contribution of nancial shocks to output and hours in the endogenous uncertainty model (black). The red, dashed line is the historical path of output and hours in the exogenous uncertainty benchmark, when it is subject to the same set of historical shocks as the endogenous uncertainty model. Dotted is the data, which di ers from the black line due to the presence of other shocks.