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D n s

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**Abstract**

This paper proposes a new explanation for the failure of Uncovered Interest Parity

## Introduction

Standard international models imply that the returns on default-free deposits across currencies should be equal. This is known as the Uncovered Interest Parity (UIP) condition and it plays a central role in exchange rate determination in most models. Yet a long-standing puzzle in the literature is that this key condition fails in the data, as there is significant forecastable variation in currency returns. The basic finding underlying the so called UIP puzzle is that an increase in the domestic interest rate relative to the foreign one is associated with an increase in the excess return on the domestic over the foreign currency.<sup>1</sup> Moreover, recent evidence has shown that the puzzle is even more complex: the comovement between interest rate differentials and excess currency returns reverses direction at longer horizons, with high interest rates forecasting a decrease in excess currency returns at 4 to 7 year horizons.

This paper proposes a new mechanism that can rationalize both the classic UIP puzzle

bonds) changes. The bonds are issued by the governments in the two countries, who finance a fixed level of real expenditures by issuing nominal debt and levying lump-sum taxes. Monetary policy is set via a Taylor rule and tax policy via a [Leeper \(1991\)](#) rule, and the only exogenous shocks are standard productivity and monetary shocks.

In this model, excess currency returns arise as compensation for differences in the liquidity value of the two bonds, and thus equal the bonds' convenience yield differential. In equilibrium, this differential is closely tied to the relative supply of home and foreign debt. Intuitively, as one country's debt becomes relatively scarce, its convenience yield increases *relative to* the other's convenience yield, and vice versa.<sup>2</sup> To illustrate, consider a contractionary home monetary shock that increases interest rates, and lowers inflation and output. The increase in the real interest rate and fall in output (which lowers taxes) combine to increase home government debt, lowering its convenience yield re

remain relatively high even as debt falls back towards steady state. This leads home debt to overshoot and fall below steady state before converging, but as it falls below steady state it now becomes relatively scarcer than foreign debt, and thus the convenience yield differential turns positive. As a result, the compensating excess return switches to the foreign currency, and this generates a change in the direction of UIP violations at longer horizons.

I analyze the mechanism in two steps. First, I derive analytical results in a stylized version of the model that distills it to its two key ingredients: endogenous convenience yield fluctuations and the interaction of monetary and fiscal policy. There I analytically charac-

model the equilibrium return on long-term investments across countries is equal to the sum of expected future short-term convenience yield differentials. But since the convenience yield differential has cyclical dynamics and changes signs, the sum of future expected differentials is roughly zero, leading to no significant UIP violations in long-term bonds.

In terms of convenience yield research, a number of papers have quantified them in the data and documented their important role in the determination of equilibrium bond prices (e.g. [Fontaine and Garcia \(2012\)](#), [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), [Smith \(2012\)](#), [Greenwood and Vayanos \(2014\)](#)). A related theoretical literature has explored bond convenience yields as a possible explanation for closed economy asset pricing puzzles such as the equity risk-premium, the low risk-free rate and the term premium (e.g. [Bansal and Coleman \(1996\)](#), [Lagos \(2010\)](#), [Bansal et al. \(2011\)](#) respectively). I extend the theoretical analysis of convenience yields by introducing them to an open economy setting, and studying their implications about exchange rate determination. I also provide new empirical results showing that convenience yields appear to be important drivers of exchange rates in the data.

The paper is organized as follows. Section 2 establishes the motivating empirical facts, and Section 3 introduces the idea of convenience yields. Section 4 lays out and analyzes the analytical model, while Section 5 presents the quantitative model. Sections 6 and 7 provide direct empirical evidence in support of the mechanism, and Section 8 concludes.

## E p r a E n

I begin by documenting the failure of UIP at different horizons. I use daily data on forward

rate determination in standard models. Its empirical failure, however, is one of the best





to offset it and hence earn high excess returns – this is the classic ‘UIP Puzzle’. However, notice that the coefficients change sign at longer horizons, and are actually *positive* and statistically significant at horizons between 48 and 84 months. This signifies that high interest

currency returns predictability into interest rate and exchange rate components. I find that the non-monotonicity in the returns arises because the exchange rate exhibits a particular type of 'delayed overshooting' where following an interest rate increase it appreciates initially, but then eventually experiences a pronounced period of  $x_{ss}$  depreciation that drives the positive UIP violations. Interestingly, the eventual depreciation more than offsets the initial appreciation, and in the long-run the exchange rate converges to the path implied by UIP.

To show this, I compare the actual response of the exchange rate to a change in the interest rate differential to its counter-factual path under

I estimate the needed  $\alpha_k$  coefficients with a similar fixed-effects panel regressions.

Figure 2 plots the results. The blue line plots the actual IRF,  $\hat{\alpha}_k$ , with its 95% confidence interval as the shaded area around it, and the red dash-dot line plots the UIP counterfactual. One can read the cumulative UIP violations  $(\sum_{h=1}^k \alpha_h)$  of this graph as the distance between the red and the blue line. For example, the initial diverging movements in the lines underlies the classic UIP puzzle (negative  $\alpha_k$  at short horizons). Intuitively, an increase in the interest rate generates a persistent rise in the interest differential, and hence UIP predicts that in response the exchange rate will experience a sustained depreciation – the upward sloping path of the red line. On the contrary, however, the exchange rate fails to depreciate and in fact even appreciates at horizons of up to 36 months, as we can see from the dip in the blue line. Thus, the exchange rate does not close the profit opportunities arising from the larger interest rate differential, but rather enhances them, giving rise to high excess currency returns in the short-run.

horizons, is what generated the positive  $\kappa$  UIP coefficients.<sup>10</sup>

Another way to think about the role of the exchange rate in driving the cyclical behavior of the excess return is to compare the actual path of the exchange rate to the Random Walk path (the black dashed line at zero in the figure). If the exchange rate was truly a random walk, then it would have no predictable movements and all of the predictable cyclical movements in the excess return must be coming from the interest rate differentials themselves. On the contrary, however, even though the exchange rate appears like random walk at short

Krishnamurthy and Vissing-Jorgensen (2012), Greenwood and Vayanos (2014)). The convenience yield is the amount of interest investors are willing to forego in exchange for the non-pecuniary benefits of owning high-quality debt. Those benefits arise from the high safety and liquidity of risk-free debt, which makes it a good substitute for money, a special asset that investors are willing to hold at zero interest rate. For example, Treasuries serve an important role as collateral in facilitating complex financial transactions, back deposits, and often even act as direct means of payment between financial institutions. Hence, they provide many of the special features of money as medium of exchange and store of value, and as a result share in some of its holding benefits.

In an international context, the convenience yield differential between the bonds of two countries,  $i_t - i_t^*$ , acts as a wedge in the Euler equation, such that up to first-order

$$E_t(s_{t+1} - s_t + i_t^* - i_t) = i_t - i_t^* \quad (4)$$

Hence, investors balance not only the expected relative financial return on the two bonds, but also the differences in their liquidity values. In equilibrium, currency returns would adjust to offset the convenience yield differential – when the home bond convenience yield is relatively high, investors require a higher financial return on the foreign bond as compensation, which gives rise to time-variation in excess currency returns, and violates UIP.

This is a wedge that has not been studied previously as a possible explanation of the UIP puzzle, but is a potentially important force. Empirical estimates of the average convenience yield on US Treasuries, for example, range between 75 and 166 basis points, and estimates of the standard deviation range between 45 and 115 bp.<sup>12</sup> It is a large and 9(e)3.38582056(1e86(s

test whether differences in expected monthly equity returns across countries are offset by exchange rate movements, and find that indeed they are, in contrast to the typical result of UIP tests. Thus, it appears that excess currency returns are non-zero only when transacting in assets close to money, suggesting that convenience yields could play an integral role.

To explore this hypothesis further, I develop a model with endogenous fluctuations in equilibrium convenience yields and test its key implications in the data.

## **Analytical**

I start by presenting an intentionally stylized version of the model that allows for analytical results and a clean illustration of the main mechanism. In the next section, I relax the simplifying assumptions made here, set the mechanism in a two country general equilibrium model, and show that all the insights from this section transfer fully.

assumption that home and foreign bonds are not perfect substitutes, so that

$$|u_{b_i, b_i}(\cdot)| > |u_{b_i, b_f}(\cdot)|$$

## Government

The government sets monetary policy according to a standard Taylor rule

$$\frac{(1 + i_t)}{1 + i} = (-\alpha) \pi e^{v_t}$$

where  $v_t$  is white noise. On the fiscal side, it faces a constant level of real expenditures  $g$  and the budget constraint

$$b_t^G + t = \frac{(1 + i_{t-1})}{1 + i} b_{t-1}^G + g$$

where  $b_t^G$  is real government debt. I follow the literature on the interaction of monetary and fiscal policy and assume that the lump-sum taxes are set according to the linear rule<sup>14</sup>

$$t = \tau b_{t-1}^G + (1 - \tau) g$$

where  $\tau \in [0, 1]$ ,





$b_{ft} = 0$ , and that home agents must hold the whole supply of home bonds:

$$b_{ht} = b_t^G.$$

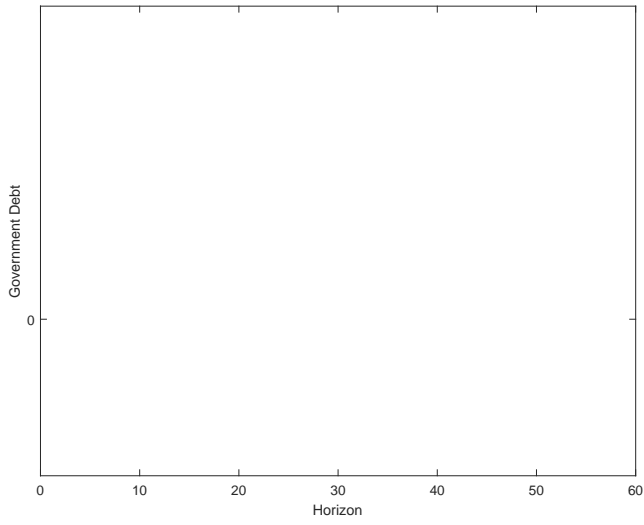






Figure 3: Debt Dynamics and UIP Violations

(a) Response to Contractionary Monetary Shock













## Firms

There is a home representative final goods firm which uses the domestic continuum of intermediate goods and the following CES technology to produce total output  $Y_{H,t}$ :

$$Y_{H,t} = \int_0^1 Y^{\xi-1}$$

bonds to fund a constant level of real expenditures ( $g$ ) and faces the budget constraint

## Calibration

The benchmark calibration is presented in Table 1, with one period in the model representing one quarter. I set risk aversion equal to 3,  $\beta = 0.9901$ , and the inverse Frisch elasticity of labor supply  $\eta = 1.5$ , all of which are standard values in the RBC literature. Estimates of the elasticity of substitution between home and foreign goods vary, but most fall in the range from 1 to 2 and I follow Chari et al. (2002) and set  $\sigma = 1.5$ . I set the elasticity of substitution between domestic goods,  $\sigma_d$ , equal to 7.66, implying markups of 15%, and choose the degree of home bias  $a_h = 0.8$ , a common value in the literature that is roughly in the middle of the range of values for the G7 countries.

In calibrating the transaction cost function, I set  $\alpha, \mu, \gamma, \bar{m}$  to match the interest rate semi-elasticity of money demand, the income elasticity of money demand, money velocity and the average convenience yield. I target an interest rate semi-elasticity of money demand of 7, roughly in the middle of most estimates, which range from 3 to 11

between home and foreign bonds, so I set it equal to 0.25 to match the US data on the volatility of foreign bond holdings to GDP. In the model, increasing  $\beta_b$  makes the home and foreign bonds better substitutes and increases the overall volatility of foreign bond holdings.

I calibrate the steady state ratio of government spending to GDP to 22% and the ratio of government debt to GDP to 50%, the average values of total federal spending to GDP and total federal debt to GDP, respectively, in US data. For the Taylor rule I set  $\alpha = 1.5$ , and pick  $\rho_i = 0.9$  to match the persistence of the US interest rate.<sup>27</sup> Lastly, I estimate the postulated tax rule using US data on federal taxes and debt, and obtain  $\alpha = 0.92$  and  $\beta_b = 0.48$ .<sup>28</sup> The Calvo parameter is set to  $\theta = 0.667$ .

For the TFP process I use  $\rho = 0.95$ ,  $\sigma = 1.25$ , and  $\mu = 0.01$ . For the monetary policy process I use  $\rho = 0.95$ ,  $\sigma = 1.25$ , and  $\mu = 0.01$ .

**Figure 4: Regression Estimates, Model vs Data**

The general equilibrium model has more moving parts than the analytical model, but the main mechanism underlying the UIP violations and the non-monotonic exchange rate dynamics is the same. Contractionary shocks, either monetary or TFP, lower inflation and increase the real interest rate, leading to a rise in the stock of real home debt. As home debt becomes less scarce, its marginal liquidity value relative to foreign debt falls and as a result the home currency earns compensating excess returns in equilibrium. This generates the classic UIP Puzzle that high interest rates today are associated with higher expected excess currency returns. In turn, the combination of active monetary policy and a sluggish tax policy delivers cyclical debt dynamics (for the same reasons as in the analytical model), and as a result the direction of the UIP violations reverses at longer horizons

A key difference with the analytical model is that here there are also international spillover effects, which were missing in the analytical model because there changes in the allocations of the (small) foreign country had no general equilibrium effects. In particular, as the home interest rate rises the home currency appreciates, leading to higher inflation and output abroad, which improves the budget situation of the foreign government and the real supply of foreign bonds falls. Thus, while home bond supply is increasing, the foreign bond supply is decreasing, which makes home debt relatively less scarce, and serves as a reinforcing effect. Quantitatively, this effect is stronger conditional on TFP shocks, but qualitatively it plays a similar role in excess return dynamics as driven by both types of shocks.

### Conclusion

For the regression results in the previous section to be fully meaningful, it is important that the model also delivers appropriate unconditional moments for the key variables. To verify this, Table 2 presents the corresponding moments, with the second column reported



Table 2: Unconditional Moments

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Data	Benchmark Model	Monetary Shocks Only	TFP Shocks Only	No Convenience Yield
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are movements in the percentage deviation from steady state (or trend in the data).<sup>34</sup>

Relatedly, the assumption that all debt is short-term debt is innocuous, and introducing long-term bonds will in fact only strengthen the results. This is again because the model is driven by log-deviations of the relevant debt variable from its trend

is due to non-monotonic dynamics and not due to a specific shock, or combination of shocks. On the other hand, [Itskhoki and Mukhin \(2016\)](#)

curve and the stock market volatility – for both the US and the relevant foreign country in each bilateral relation. Thus, I estimate

$$r_{j,t+1} = \alpha_j + \beta_1 (i_t - i_{j,t}^*) + \beta_2 \ln(\text{Debt}_t) + \beta_3 \ln(\text{Debt}_{j,t}^*) + \beta_4 \ln(\text{CP}_t) + \beta_5 \text{NFA}_t + \beta_6 \tilde{V}_t + \text{KVJ controls} + \epsilon_{j,t+1}$$

as a panel regression with fixed effects. Following the equilibrium condition of the model, eq. (17), I include the debt variables in real terms, after removing a deterministic exponential time-trend. However, as a robustness check, I also re-estimate all specifications using debt-to-GDP ratios instead, and all results remain the same – please see Appendix E for details.

Due to availability of data on quarterly foreign debt, the sample for this analysis starts in 1991. With the exception of the Deutsche Mark (which series has the EUR appended to it at the end), this leaves the Euro legacy countries with a short sample size of at most 8 years of data (differing slightly due to government debt availability), and hence I drop them from the benchmark specification. Thus, the data for the benchmark results spans 1991-2013 for the 10 non-Euro currencies, including the German Deutsche Mark.<sup>36</sup> However Appendix E shows that the results are robust to extending the sample - there I re-estimate all regressions omitting foreign debt, which allows me to extend the sample to 1984.

Table 3 reports the estimation results. In the left panel, I report estimation results on the whole sample, which includes both the financial crisis and the post-crisis zero interest rate environment. There is good reason to believe that this latter part of the sample is a period in which the convenience yield mechanism is not very strong. In the current zero interest rate environment, liquidity needs are fairly well satiated and the convenience yield is near-zero, while during the peak of the crisis period excess returns were likely predominantly driven by risk-premium considerations. To explore this potential difference, in the right panel I report estimation results excluding the crisis and the subsequent period.

The results in both panels strongly support the model, but indeed the support is especially strong in the pre-crisis period. In all specifications, the coefficient on US debt is negative and significant, which signifies that just like in the model, in the data times of higher US government debt are associated with higher excess returns on the USD. The estimates are also economically significant, as they imply that a one standard deviation increase in US debt is associated with a 60bp increase in the (monthly) excess return on the USD. This is a stronger effect than the corresponding relationship with the interest rate differential (as

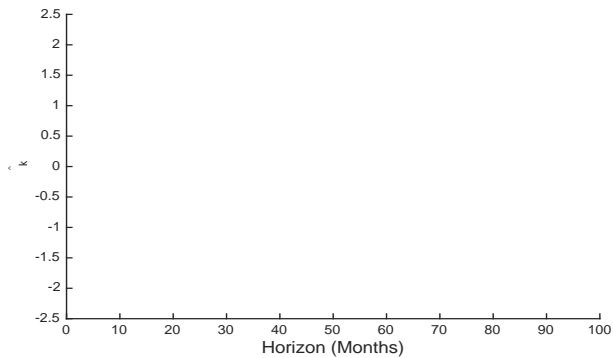
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<sup>36</sup> To maximize the data and keep as close as possible to the original empirical analysis in Section 2.1, I consider 1-month excess currency returns at the daily frequency. I use quarterly debt to create daily frequency debt series, by using last quarter's debt to fill-in the daily values for the current quarter. Thus, the debt observation for March 31 is used for all days in April, May and June. This avoids look ahead bias, and ensures that the regressors contain at most time  $t$  information. As a robustness check, I re-estimate all specifications at the quarterly frequency and the results remain the same – for details see Appendix E.



a lot of the explanatory power of the classic UIP regression is attributable to the omitted debt variables, as suggested by the model. Moreover, introducing the debt controls also leads to an economically significant improvement in the  $R^2$  of the regressions. The interest rate differential by itself is able to muster only a (within)  $R^2$  of 0.014, while adding the debt controls more than triples that value to 0.043. Alternatively, we can ask how much of the specific currency excess return captured by the forecasting power of the interest rate differential is explained by the supply of debt. To answer that, I first project the realized currency returns on the interest rate differential, and then regress the predicted returns,  $\hat{r}_{t+1} = (i_t - i_t^*)$ , on the debt variables. The second stage regression yields a  $R^2$  of 0.37, suggesting that the convenience yield mechanism is able to explain almost 40% of the classic UIP puzzle. Hence, I conclude that the effect of the supply of debt, a sufficient statistic in the model, is both statistically and economically significant.

Figure 6: Excess Currency Returns and Debt at All Horizons





regression (eq. (2)) with the debt variables considered in this section and plot the resulting coefficients in Figure 6. Due to the shortened sample, I consider  $k = 100$  months.

Several interesting results emerge. First, the top left panel sho

and regress the excess currency returns on it, while also controlling for the level of NFA and the signed VIX index.

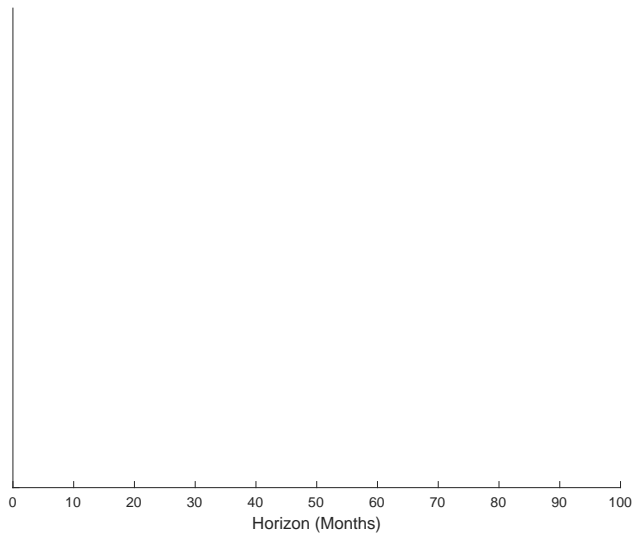
Table 4: Excess Currency Returns and Conv. Yields, 1991 - 2013

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	(1)	(2)	(3)	(4)	(5)	(6)
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Figure 7: Convenience Yield and Excess Currency Returns at All Horizons

(a) AAA-Treasury



and further sort them on their fiscal policy in two ways. First, I compute the autocorrelation of the growth in public debt, which is positive when taxes are relatively sluggish and debt displays non-monotonic dynamics. Second, I directly estimate the tax policy rule posited by the model, compute the implied threshold value  $\bar{b}$  as per Lemma 2 and check which countries have estimates above that threshold. Only three countries meet those criteria – CAD, GBP and USD. Re-estimating the UIP regression with the six currencies with strong monetary policy as alternative base currencies, I find that only the three currencies with

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# International Application For Information

## A Data Description

The data set consists of forward and spot exchange rates from Reuters/WMR and Barclays, and is available on Datastream. It includes the Euro and the currencies of the following 18 advanced OECD countries: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland and the UK.

The data spans the time period 1976:M1-2013:M6 and is at a daily frequency. The data on the Euro-legacy currencies (e.g. France, Austria, etc.), except for the German Deutsch

where lower case letters represent variables in logs and I have used the approximation  $i_t \approx \ln(1 + i_t)$ .<sup>38</sup> Thus, up to a first-order approximation, the expected return on foreign bonds,  $E_t(s_{t+1} - s_t + i_t^*)$ , equals the expected return on the home bond,  $i_t$ . This restricts the joint dynamics of exchange rates and interest rates, and delivers strong implications for exchange rate behavior. The condition obtains in a large class of standard open economy models.

B  
Ca ss 

The failure of the UIP condition in the data is a long-standing and well documented puzzle in international finance, with a large and still active literature expanding on the seminal contributions by B.

West standard errors. The results are reported in Table B.1, and the estimates reaffirm the well established UIP Puzzle - I find that all  $\beta_1$  point estimates are negative and almost all are statistically significant at conventional levels (15 out of 18). The evidence of negative and significant  $\beta_1$  is remarkably consistent throughout all 18 currencies. Estimating equation (B.3) as a panel regression, where  $\beta_1$  is restricted to be the same for all currency yields a significantly negative coefficient as well.

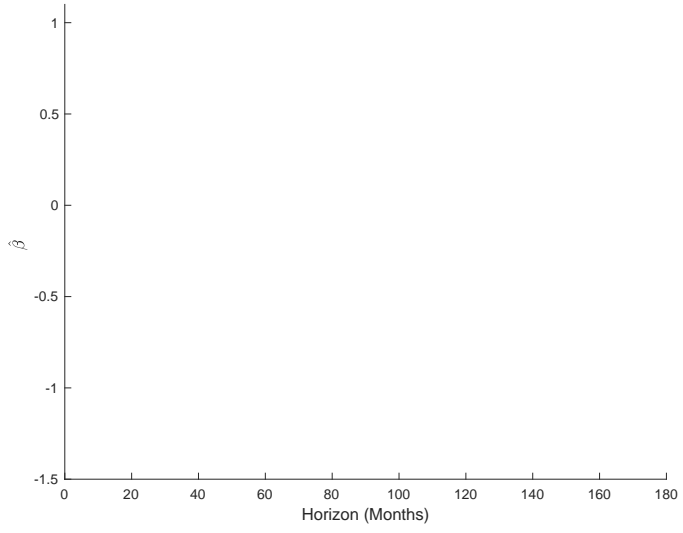
Table B.1: UIP Regression Currency by Currency

Country	Currency	$\beta_0$	(s.e.)	$\beta_1$	(s.e.)	$\chi^2(\beta_0 = \beta_1 = 0)$	R <sup>2</sup>
Australia	AUD	-0.001	(0.002)	-1.63***	(0.48)	16.3***	0.014
Austria	ATS	0.002	(0.002)	-1.75***	(0.58)	9.5***	0.023
Belgium	BEF	-0.0002	(0.002)	-1.58***	(0.39)	17.5***	0.025
Canada	CAD	-0.003	(0.001)	-1.43***	(0.38)	19.1***	0.013
Denmark	DKK	-0.001	(0.001)	-1.51***	(0.32)	25.4***	0.025
France	FRF	-0.001	(0.002)	-0.84	(0.63)	1.9	0.007
Germany	DEM	0.002	(0.001)	-1.58***	(0.57)	7.9**	0.015
Ireland	IEP	-0.002	(0.002)	-1.32***	(0.38)	12.3***	0.020
Italy	ITL	-0.002	(0.002)	-0.79**	(0.33)	7.0**	0.013
Japan	JPY	0.006***	(0.002)	-2.76***	(0.51)	28.9***	0.038
Netherlands	NLG	0.003	(0.002)	-2.34***	(0.59)	16.0***	0.041
Norway	NOK	-0.0003	(0.001)	-1.15***	(0.39)	10.4***	0.013
New Zealand	NZD	-0.001	(0.002)	-1.74***	(0.39)	28.3***	0.038
Portugal	PTE	-0.002	(0.002)	-0.45**	(0.20)	5.9*	0.019
Spain	ESP	0.002	(0.003)	-0.19	(0.46)	2.8	0.001
Sweden	SEK	0.0001	(0.001)	-0.42	(0.50)	0.9	0.002
Switzerland	CHF	0.005***	(0.002)	-2.06***	(0.55)	13.9***	0.026
UK	GBP	-0.003**	(0.001)	-2.24***	(0.60)	14.2***	0.028
Panel, pooled		0.0002	(0.001)	-0.79***	(0.15)	22.3***	
Panel, fixed effects				-1.01***	(0.21)	19.1***	

This table presents estimates of  $\beta_0$  and  $\beta_1$  from the regression  $s_{j,t+1} - s_{j,t} + i_{j,t}^* - i_{j,t} = \beta_0 + \beta_1(i_{j,t} - i_{j,t}^*) + \epsilon_{j,t+1}$ . The standard errors in single currency regressions are Newey-West errors robust to serial correlation. The standard errors for the panel estimations are computed according to the Driscoll and Kraay (1998) method that is robust to heteroskedasticity, serial correlation and contemporaneous correlation across equations. The base currency is the USD.

**Figure B.1: Regression Results on pre-2008 Sample**

**(a) UIP Violations**



So as we can see, the predictability in the excess currency returns at horizons of over 36 months is almost exclusively due to predictability in exchange rate changes. In particular, at these longer horizons the exchange rate is expected to sustain a significant depreciation (positive  $\tilde{\kappa}$ ), which results in negative expected excess currency returns at those horizons.

In conclusion, the results of this section confirm that the change in the sign of the excess return predictability (the sign on the  $\kappa$  coefficients) is driven by a change in the sign of the predictability in high frequency exchange rate movements at longer horizons. This complements the discussion in Section 2.2 which argues that it is the changing nature of exchange rate predictability that underlies the estimated cyclicity of the currency excess returns.

C      r o o s

equal the supply of home government debt, the system of equilibrium conditions becomes

$$\hat{i}_t = E_t(\hat{r}_{t+1}) + \hat{b}_t - M(E_t(\hat{b}_{h,t+1}) - \hat{b}_{h,t})$$
$$\hat{b}_{ht} + \frac{\hat{r}_t}{b_h} = (1 + i$$



$$-(b) = \frac{b(b - ) + (b + )^2 + 2 \sqrt{b^2(b - + 2)}}{(2 + b)^2}$$

Since  $2 <$  it follows that  $-(b) < 1$  and since

$$b(b - ) + (b + )^2 = b(b - + 2) + 2$$

it follows that  $-($

On the other hand, if  $b = + 2 \tau$

$$\left( - (1 - ) b + 2 \right)^2 - 4 \quad 0$$

and hence the eigenvalues are always real. Moreover, above we showed that when the eigenvalues are real,  $\frac{1}{b} <$

$$\hat{\pi}_t = \hat{b}_{ht} = 0$$

Next, we can substitute this result in the government budget and obtain the relationship

$$i_{t-1} = \pi_t$$

Substituting in the Taylor rule we find the solution for inflation:

$$\pi_t = \pi_{t-1} + v_{t-1}$$

Since  $\lambda < 1$ , this is stationary and this concludes the forward direction of the proof. We have shown that when either conditions (i) or (ii) are satisfied, there is a determinate stationary equilibrium.

In proving the necessary direction, I start with the case where  $\lambda > 1$ . This time I will first deal with the conditions on  $b$ , and to this end assume that  $b < -2$ . Above we

showed that in this case the roots are always real, and that  $\lambda_1 = 1$ , and that  $\frac{-1}{\lambda} < 0$

for  $\lambda < \frac{1+\chi_2}{1-\tau}$  which holds since  $-2 < \frac{1+\chi_2}{1-\tau}$ . Therefore, it is immediate that  $b < -2$  leads to a root bigger than one and thus explosive solutions.

On the other hand if  $b > \frac{(1+\chi_2)(1-\tau)}{1-\tau}$ , then

$$\left( - (1 - \lambda) b + \lambda \right)^2 - 4 \lambda > 0$$

so the roots are again always real. Moreover, we have already shown that  $\lambda_2 = \frac{(\theta - \theta_2)(1 + \rho\tau)}{1 - \rho\tau} =$

$-1$ , and that  $\frac{-2}{b} < 0$  for  $b > \frac{(-2)(1 + \tau)}{1 - \tau}$ , hence it follows that  $|\lambda_2| > 1$  for all  $b > \frac{(-2)(1 + \tau)}{1 - \tau}$ , and thus we again have an explosive root.

Next, turn attention to  $\lambda > 2$  and  $b < \left( -2, \frac{(-2)(1 + \tau)}{1 - \tau} \right)$ . If  $\lambda > 2$ ,

Next, I treat the case  $\tau < 1$ . If  $b \in \left[ -2, \frac{(-2)(1+\tau)}{1-\tau} \right)$

$$x_t = Bv_t + ABv_{t-1} + A^2Bv_{t-2} + \dots$$

and use the fact that

$$B = \frac{1+i}{\pi} v_t$$

to obtain

$$\hat{b}_{ht} = \frac{1+i}{\pi} (v_t + a_{11}^{(1)}v_{t-1} + a_{11}^{(2)}v_{t-2} + a_{11}^{(3)}v_{t-3} + \dots)$$

$$\hat{t} = \frac{1+i}{\pi} (a_{21}^{(1)}v_{t-1} + a_{21}^{(2)}v_{t-2} + a_{21}^{(3)}v_{t-3} + \dots)$$

where  $a_{lm}^{(k)}$  is the  $(l,m)$  element of the matrix  $A^k$ . Define  $a_{11}^{(0)} = 1$  and  $a_{21}^{(0)} = 0$  and the transformation

$$a_{bk} = \frac{1+i}{\pi} a_{11}^{(k)}$$

The sequence  $\{a_{bk}\}_{k=0}^{\infty}$  defines the Impulse Response Functions of  $\hat{b}_{ht}$ .

First, I will show that  $a_{bk} = 0$  for all  $k = 0, 1, 2, \dots$  when the matrix  $A$  is diagonalizable, and then I will handle the case when the eigenvalue is repeated and  $A$  is not diagonalizable (the only other case we need to worry about for a two by two matrix).

Assuming that  $A$  is diagonalizable, define

$$= \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

as a matrix with the two eigenvalues of  $A$  on its diagonal ordered like  $\lambda_1 > \lambda_2$  (remember we are handling the case of real eigenvalues right now) and  $P$  as a matrix that has the eigenvectors of  $A$  as its columns. Since we have assumed  $A$  is diagonalizable, we have  $A = P \Lambda P^{-1}$  and also  $A^k = P \Lambda^k P^{-1}$ . Since  $\Lambda$  is diagonal

$$\Lambda^k = \begin{pmatrix} \lambda_1^k & 0 \\ 0 & \lambda_2^k \end{pmatrix}$$

and thus if we expand the expression for  $A^k$  we obtain that

$$a_{11}^{(k)} = \frac{p_{11}p_{22} \lambda_1^k - p_{12}p_{21} \lambda_2^k}{|P|}$$

where  $|P|$  is the determinant of the matrix of eigenvectors  $P$  and  $p_{lm}$  is its  $(l,m)$ -th element. Since both of the eigenvalues are positive and are ordered so that  $\lambda_1 > \lambda_2$  it follows that  $|P| > 0$  and hence

$$\frac{p_{11}p_{22}^k - p_{12}p_{21}^k}{|P|} > 0.$$

This proves that  $a_{11}^{(k)} > 0$  for all  $k$  and hence  $a_{bk} > 0$  for all  $k$ . This completes the proof for diagonalizable  $A$  – now assume that  $A$  is not diagonalizable. We can instead use the Jordan Decomposition to again write  $A = P \Lambda P^{-1}$  but now

$$\Lambda = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$$

and the columns of  $P$  are the generalized eigenvectors of  $A$ . In this case, there is only one linearly independent eigenvector associated with the eigenvalue of  $1$ , call it  $p$ , and thus the second generalized eigenvector, call it  $u$ , is a  $2 \times 1$  vector that solves

$$(A - I)u = p$$

We can solve for the needed eigenvectors via standard techniques, and obtain  $p = [p_1, 1]'$  and  $u = [u_1, 1]'$ , where  $p_1 = \frac{-\tau}{(1-\tau)b} + \frac{\tau}{b_h}$ ,  $u_1 = p_1 + \frac{1}{(1-\tau)b} + \frac{\tau}{b_h}$ . We can then use  $A^k = P \Lambda^k P^{-1}$  to get:

$$a_{11}^{(k)} = \lambda^{k-1} \left( p_1 + k \frac{p_1}{u_1 - p_1} \right) > 0$$

The inequality follows from  $u_1 > p_1 > 0$ ,  $\tau > 0$ . This completes the proof of part (i).  
 From the proof of Lemma 1 we know that  $(\tau, \tau)$  implies that the eigenvalues of  $A$  are complex. We can express them as  $\lambda_1 = a + bi$  and  $\lambda_2 = a - bi$  where  $a = \frac{1}{2}(-\tau - (1-\tau)b + \tau) > 0$ ,  $b = \frac{1}{2}\sqrt{4\tau^2 - (-\tau - (1-\tau)b + \tau)^2} > 0$  and  $i$  is the imaginary unit. The two conjugate eigenvectors can be written as  $p_k = [x \pm yi, 1]'$ , where

$$x = \frac{(-\tau - (1-\tau)b + \tau - 2\tau)}{2(1-\tau)b}$$

$$y = \frac{\sqrt{4\tau^2 - (-\tau - (1-\tau)b + \tau)^2}}{2b(1-\tau)}$$

With two conjugate complex eigenvalues  $A$  is diagonalizable and can be expressed as  $A = P \Lambda P^{-1}$  where  $P$  is a similarity matrix with the eigenvectors of  $A$  as its columns and  $\Lambda$  is a diagonal matrix with the eigenvalues on the diagonal. By Euler's formula  $\lambda_1 = a + bi = |e^{i\theta}|$  where  $\theta = \arctan(\frac{b}{a})$  and  $|e^{i\theta}| = \sqrt{a^2 + b^2}$  is the magnitude of the complex roots. This formulation is convenient because it is easy to take powers of the eigenvalues, (e.g.  $\lambda_1^k = |e^{i\theta}|^k e^{ik\theta}$ ) and hence it is easy to compute powers of the eigenvalue matrix  $\Lambda$ . Using this, Euler's formula and the fact that  $A^k = P \Lambda^k P^{-1}$  it is straightforward to compute

$a^{(k)}$





Thus,

$$\begin{aligned}\hat{i}_t - \hat{i}_t^* &= (\hat{b}_{ht} + \hat{t}) \\ &= ((\hat{a}_{b0} + \hat{a}_0)v_t + (\hat{a}_{b1} + \hat{a}_1)v_{t-1} + \dots) \\ &= \hat{a}_{i0}v_t + \hat{a}_{i1}v_{t-1} + \hat{a}_{i2}v_{t-2} + \dots\end{aligned}$$

On the other hand, if  $b = +2$

coefficients as

$$\beta_k = \frac{\text{Cov}(-\beta E_t(\hat{b}_{n,t+k-1}), (\beta \hat{b}_{nt} + \hat{\epsilon}_t))}{\text{Var}(\beta \hat{b}_{nt} + \hat{\epsilon}_t)} = -\beta \left( \beta \frac{\text{Cov}(E_t(\hat{b}_{n,t+k-1}), \hat{b}_{nt})}{\text{Var}(\beta \hat{b}_{nt} + \hat{\epsilon}_t)} + \frac{\text{Cov}(E_t(\hat{b}_{n,t+k-1}), \hat{\epsilon}_t)}{\text{Var}(\beta \hat{b}_{nt} + \hat{\epsilon}_t)} \right)$$

Since  $E_t(\hat{b}_{t+k}) = [1, 0]$

$2 \frac{b_h}{1+\tau} \text{Var}(\hat{b}_{ht})$ , and hence the UIP coefficient becomes

$$k_{+1} = -\frac{b}{\text{Var}(\hat{i}_t)} \left( b(a_{11}^{(k)} + a_{12}^{(k)}) + (a_{11}^{(k)} + a_{12}^{(k)}) \left(\frac{b_h}{1+\tau}\right)^2 \frac{2(1-\tau)}{1+\tau} \right)$$

At  $b = -2$ , the expression becomes

$$(1 - \alpha)(2 + \alpha)(2 - \alpha) > (1 - \alpha)(2 + \alpha)(2 - \alpha) > 0$$

where the first inequality follows from  $\alpha < \frac{2}{3}$ , and the second from  $\alpha > 1$ .

On the other hand, its derivative at  $b = -2$  is:

$$\begin{aligned} & ((1 - \alpha)^2 + 2(\alpha - 2(1 - \alpha))) + M(1 - \alpha)(2 + 1 - \alpha) + 2i(\alpha - 2) + 2(1 - \alpha)(1 - \alpha - 2) \\ & > (1 - \alpha)^2(M + 2 + (\alpha + M)) \\ & > 0 \end{aligned}$$

where the first inequality follows from the fact that the top line is increasing in  $\alpha$  and  $\alpha > 1$ . Thus, we have shown that (C.7) is positive and increasing at  $b = -2$ , and hence  $b + \alpha > 0$  which implies that  $\alpha_1 > 0$ . This completes the proof of part (i), sub-point b.

By the proof of Lemma 3 the eigenvalues of  $A$  are 1. Tre-167.65 Td[(1)4.03117(in)0.

in the pre-2008 period. The opening up of a persistent CIP deviation in the latter part of





As we saw in the main text, contractionary shocks increase  $i_t$  while lowering  $\hat{i}_t^H$  – this is the key feature generating the UIP Puzzle, since it leads to the result that high interest rates are associated with high excess currency returns (which compensate for the low  $\hat{i}_t^H$ ). However, this exact same mechanism also leads to an increase in  $\tilde{i}_t$ , which generates a positive correlation between  $i_t$  and  $\tilde{i}_t$ . Lastly, the convenience yield is considerably less volatile than the Treasury interest rate itself – the std deviation of  $\hat{i}_t^H$  is only half of that of  $i_t$ . These forces together result in a high, positive correlation between  $i_t$  and  $\tilde{i}_t$ .

Thus, the bottom line is that the model implies that the interest rates on different types of assets, some more liquid than others, will be highly correlated and overall behave very similarly. Just like what we observe in the data.

## Domestic Bonds

It is well known that the UIP holds better in the “long-run”. Specifically, [Chinn \(2006\)](#) and others have shown that 5-year (and longer) excess currency returns display smaller UIP deviations, than the typical estimates of the UIP Puzzle in short-term bonds. It is important to note that the model can match this observation, even if we make the strong (and counterfactual) assumption that long-term bonds are perfect substitutes for short-term bonds in terms of liquidity, and hence earn the *sa* convenience yield.

The key empirical result centers on the regression

$$S_{t+N} - S_t + R_t^{*(N)} - R_t^{(N)} = \alpha^{(N)} + \beta^{(N)}(R_t^{(N)} - R_t^{*(N)}) + \epsilon_{t+N}^{(N)}$$

where the  $R_t^{(N)} = \sum_{i=0}^{N-1} i_t^{(N)}$  is the cumulative interest rate on a N–period bond ( $i_t^{(N)}$  is the yield on the N-period bond). The left-hand variable is the excess return on N-period foreign bond over a N-period home bond when both are held to maturity. It turns out, that while  $\alpha^{(N)}$  is large and significantly negative for N = 1 years, the estimates are smaller and often insignificant for N > 1

and  $\mathbf{b}_t^{*,T}$

$$\mathbf{b}_t^T = \mathbf{b}_t^{(1)} + \mathbf{b}_t^{(2)} + \dots$$

from long-term bonds and term structure effects, as I do in the model, is unlikely to be important.

According to the EH, cumulative long-term interest rates are equal to the sum of expected future short-rates over the duration of the long-term interest rate. This implies that a zero coupon n-month bond's cumulative interest rate,  $R_t^{(n)}$ , is given by

$$R_t^{(n)} = \sum_{k=0}^{n-1} E_t(i_{t+k}),$$

where, as before,  $i_t$  is the 1-month interest rate at time  $t$ . We can then use this relation to back out risk-neutral expectations of future short-rates from the term-structure itself. Let  $i_{t,t+k}$  be the time- $t$  risk-neutral expectation of the 1-month interest rate at time  $t+k$ , also known as the *forward* interest rate at time  $t$ , and note that this is given by the difference in interest rates of a  $(k+1)$ -months bond and a  $k$ -months bond:

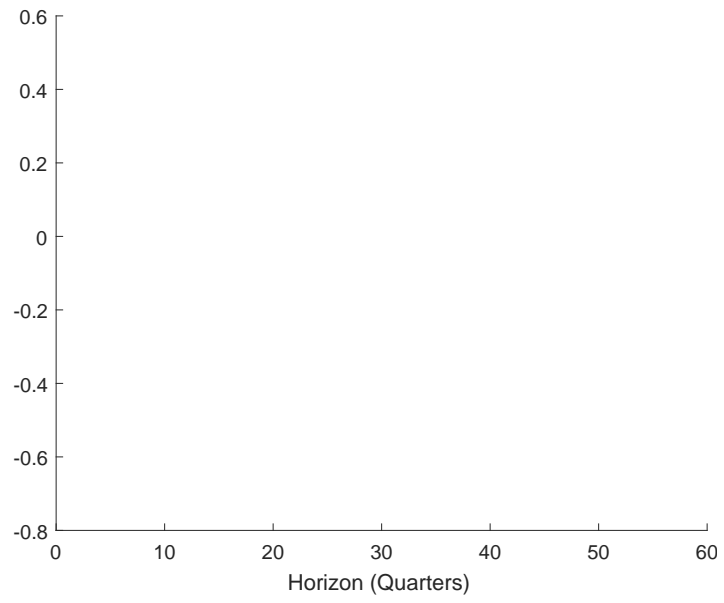
$$i_{t,t+k} = R_t^{(k+1)} - R_t^{(k)}$$



the term-structure effects are also non-zero and switch from negative to positive, their timing



Figure D.3: Regression Estimates, Real vs Nominal



to positive excess returns on the domestic currency going forward (as evidenced by  $\beta < 0$ ). These are the two main puzzling facts about real exchange rates singled out by Engel (2016) – that high real interest rates are associated with both an appreciated currency, and one that is expected to earn positive excess returns. My model is able to generate both.

One weakness, however, is that the empirical estimate in Engel (2016) calls for a much larger  $\beta \approx -40$ . There are two issues here. First, my model does not produce real interest rate differentials that are quite as persistent as those found in the data. Since

$$q_t = \sum_{k=0}^{\infty} E_t(r_{t+k}^* - r_{t+k}) - \sum_{k=0}^{\infty} E_t(\pi_{t+k+1})$$

we can see that higher persistence of the real interest rate differentials is directly linked to a stronger response by the level of the real exchange rate. The reason that the model implies lower persistence in real interest rates is most likely that the real side of the model is kept intentionally simple and free of additional frictions and mechanisms in order to highlight the role of the convenience yields in determining equilibrium exchange rate dynamics. Apart from the convenience yield mechanism and endogenous fiscal policy, this is the simplest possible two country model. I believe that adding some of the mechanisms proposed by the literature to produce more realistic inflation and interest rate differentials, such as for example local currency pricing or non-tradable goods, would help the model in this direction.

Second, the model also implies a relatively small elasticity of the sum of excess returns,  $\sum_{k=0}^{\infty} E_t(\pi_{t+k+1})$ , to real interest rate differentials, while Engel (2016) finds a large one. Still, it is notable that in my own empirical analysis (Section 2.1) I find that the elasticity of  $\sum_{k=0}^{\infty} E_t(\pi_{t+k+1})$  to nominal interest rates is quite low, and roughly zero. It might be interesting to dig further into this issue to determine a robust target for this elasticity. Nevertheless, at this stage, the model similarly implies a cumulative e





Table 1: Excess Currency Returns and Debt-to-GDP

	1991 - 2013					1991 - 2007				
	(1)	(2)	(3)	(4)	(5)	(1')	(2')	(3')	(4')	(5')
$i_t - i_t^*$	-1.4*** (0.46)	-1.55*** (0.46)	-0.86* (0.52)	-1.11 (0.83)	-1.17 (0.88)	-1.83*** (0.49)	-1.95*** (0.49)	-0.83 (0.51)	-0.47 (0.52)	-0.55 (0.52)
$\ln(\frac{Debt}{GDP})$		-0.48 (0.38)	-3.28*** (1.22)	-5.62*** (1.52)	-5.49*** (1.59)		-1.64*** (0.59)	-5.69*** (1.41)	-5.00*** (1.89)	-5.34*** (1.93)
$\ln(\frac{Debt^*}{GDP})$		0.18 (0.11)	0.27** (0.12)	0.18 (0.13)	0.21 (0.13)		0.08 (0.17)	0.12 (0.11)	0.22** (0.11)	0.16 (0.11)
$\ln(\frac{CP}{GDP})$			-2.76** (1.12)	-5.04*** (1.51)	-4.63*** (1.57)			-3.52*** (1.08)	-2.28 (1.84)	-3.02 (1.85)
NFA					0.68*** (0.20)					1.07*** (0.34)
VIX					0.27 (0.42)					0.37 (0.37)
KVJ2012 Controls	No	No	No	Yes	Yes	No	No	No	Yes	Yes
# Currencies	10	10	10	10	10	10	10	10	10	10
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Estimates with [Driscoll and Kraay \(1998\)](#)







**Figure 1: UIP Violations and Monetary Policy**

**(a) Top vs Bottom Third**



Figure 2: UIP Regressions, 1 to 180 months

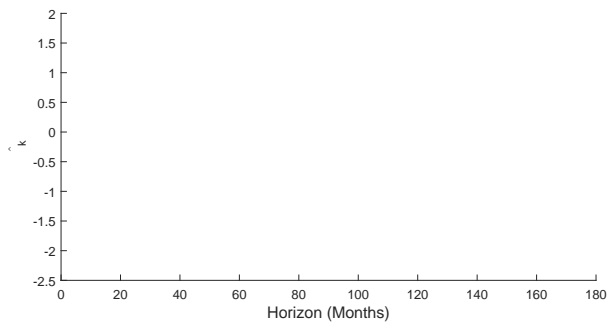


Figure 3: UIP ..... months

