CAN ECONOMIC THEORY BE INFORMATIVE FOR THE JUDICIARY? AFFIRMATIVE ACTION IN INDIA VIA VERTICAL AND HORIZONTAL RESERVATIONS

TAYFUN SÖNMEZ AND M. BUMIN YENMEZ

ABSTRACT. Sanctioned by its constitution, India is home to the world's most comprehensive affirmative action program, where historically discriminated groups are protected with vertical reservations implemented as "set asides," and other disadvantaged groups are protected with horizontal reservations implemented as "minimum guarantees." A mechanism mandated by the Supreme Court in 1995 suffers from important anomalies, triggering countless litigations in India. Foretelling a recent reform correcting the flawed mechanism, we propose the 2SMG mechanism that resolves all anomalies, and characterize it with desiderata reflecting laws of India. Subsequently rediscovered with a high court judgment and enforced in Gujarat, 2SMG is also endorsed by *Saurav Yadav v. State of UP (2020)*, in a Supreme Court ruling that rescinded the flawed mechanism. While not explicitly enforced, 2SMG is indirectly enforced for an important subclass of applications in India, because no other mechanism satisfies the new mandates of the Supreme Court.

Keywords: Market design, matching, affirmative action, vertical reservation, horizontal reservation

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has been later lled in Anil Kumar Gupta v. State of U.P. (1995), another judgment of the Supreme Court, where an explicit procedure for the concurrent implementation of VR and HR policies is devised and enforced in India. [3] For the past quarter century, this judgment has served as a main reference for virtually all subsequent litigations on concurrent implementation of VR and HR policies, of which there are thousands. This is our starting point, where our original motivations in writing this paper were; (i) formulating an important aw in the procedure mandated under this judgment, (ii) documenting its adverse consequences in India, and (iii) advocating for an alternative procedure as a remedy.

The procedure enforced under Anil Kumar Gupta (1995) rst derives a tentative outcome using the over-and-above choice rule, then it makes any necessary replacements for the tentative recipients of open positions to accommodate HR protections within open positions, and nally it makes any necessary replacements for the tentative recipients of the VR-protected positions to accommodate HR protections within VR-protected positions. We refer to this procedure as the SCI-AKG choice rule. One critical mandate in this judgment, however, has introduced two related and highly consequential anomalies into the procedure, often generating unintuitive outcomes at odds with the philosophy of af rmative action, and thereby sparking thousands of litigations in India for the next 25 years. To present the scale of the resulting disarray, some of the key litigations triggered by the awed mandate are documented in detail in Section C of the Online Appendix. [4]

The root cause of the failure of the SCI-AKG choice rule boils down to its exclusion of the members of VR-protected classes from any replacements necessary to accommodate the HR protections for open positions. Thus, members of the higher-privilege general category i.e. individuals who are not members of the VR-protected classes, are the only ones entitled to replace the tentative holders of the open positions to accommodate its HR protections. This restriction regularly created situations in India where higher-merit individuals from VR-protected classes lose their positions to lower-merit individuals from the higher-privilege general category, an anomaly we refer to as a failure of no justi ed envy. The same aw also created a con ict for individuals who qualify for both types of protections, since for these individuals claiming their VR protections would mean giving up their HR protections for open positions, an anomaly we refer to as a failure of incentive compati52 r17.3051incentive

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(4) Clarity is brought to which positions awarded to members of VR-protected classes are to be used up from open positions, rather than the VR-protected positions.

Apart from correcting a flawed mandate from *Anil Kumar Gupta (1995)* with highly disruptive consequences,[7] *Saurav Yadav (2020)* brings clarity on the meaning of VR protections in the presence of HR protections, at a level that was never done before. The defining characteristic of the VR protections is originally formulated in *Indra Sawhney (1992)* as follows:

characterization of the 2SMG choice rule with axioms that directly formulate the mandates in Saurav Yadav (2020) and what it means for India, i.e. the observation that the 2SMG choice rule in indirectly enforced with this judgment.

1.3. Extended Analysis and Policy Advice for Overlapping Horizontal Reservations.

In contrast to field applications with non-overlapping horizontal reservations where Saurav Yadav (2020) has a very sharp policy implication, as has its predecessors, the judgment leaves some flexibility for applications with overlapping horizontal reservations. In this version of the problem, an individual can benefit from multiple HR protections. We make our most significant conceptual and theoretical contributions for this general version of the problem.

Consider an individual who is a member of multiple groups, each of which is eligible for HR protections. For example, a woman with a disability can benefit from HR protections both for women and also for persons with disabilities. The law does not specify whether this individual accommodates the minimum guarantees for all HR protections she is qualified for, in this example both for women and for persons with disabilities, or for only one of them. We refer to the first convention as one-to-all HR matching and the second convention as one-to-one HR matching. While the law is silent on this aspect of the problem, we advocate for the one-to-one HR matching convention for two reasons. The first reason is technical: Adopting the alternative one-to-all HR matching convention introduces complementarities between individuals, which in turn renders the problem computationally hard in general and allows for multiplicities. In the above example, the admission of a man with no disability may depend on the admission of a woman with disability. The second reason is practical: In many real-life applications in India, the number of positions are announced for vertical category-horizontal trait pairs, which automatically embeds the one-to-one HR matching convention into the problem.

Under the one-to-one HR matching convention, an additional matching problem is essentially built into the original problem, where a secondary task matches individuals to different types of HR protections to account for these protections. Fortunately, this secondary task can be formulated as a maximum bipartite matching problem, a well-studied problem in the combinatorial optimization literature. Moreover, this approach not only allows us to formulate natural and immediate extensions of all four axioms, but also allows for a natural extension of the 2SMG choice rule in the two-step meritorious horizontal (2SMH) choice rule. In our main theoretical result (Theorem 3), we extend our characterization of the 2SMG choice rule for the case of non-overlapping horizontal reservations to the 2SMH choice rule for the general case of overlapping horizontal reservations.

1.4. Organization of the Rest of the Paper. After introducing the model in Section 2, we present analysis and policy implications for problems with non-overlapping horizontal reservations in Section 3. The failure of the mechanism mandated by Anil Kumar Gupta (1995) as the set of individuals in I who are members of the reserve-eligible category $c \in \mathcal{R}$. Given a set of individuals $I \subseteq \mathcal{I}$, define

$$I^g = \{i \in I : r(i) = \emptyset\}$$

as the set of individuals in I who are members of the general category g.

There are q^c positions exclusively set aside for the members of category $c \in \mathcal{R}$. We refer to these positions as *category-c positions*. In contrast, members of the general category do not receive any special provisions under the VR policies. Therefore,

$$q^o = q - \sum_{c \in \mathcal{R}} q^c$$

positions are open for all individuals. We refer to these positions as *open-category positions* (or *category-o positions*). Let $\mathcal{V} = \mathcal{R} \cup \{o\}$ denote the set of

HR protections are provided within each vertical category. [9] For any reserve-eligible category

It is important to emphasize that the formulation of the SCI-AKG choice rule given above is not the original formulation presented in Anil Kumar Gupta (1995). The original

Therefore, the set of individuals who are each awarded a position under the SCI-AKG choice rule is $\widehat{C}^{SCI}(I) = \{m_1^g, w_1^g, m_1^c\}$.

There are two troubling aspects of this outcome. The first issue is that, even though the category-c woman w_1^c has a higher merit score than the general category woman w_1^g, the latter receives a position while the former does not. That is, contrary to the philosophy of affirmative action, a lower merit score individual from the (unprotected) general category receives a position at the expense of a higher merit score individual from a protected category. The second issue is that, since she is the highest merit score woman among all applicants, woman w_1^c can receive the open-category HR-protected position for women simply by not declaring her eligibility for the VR-protected position for category-c.

Definition 6. Given a vertical category $v \in V$, the (category-v) *HR-maximality function* $n^v : 2^{I^v} \to \mathbb{N}$ is defined as, for any I

Eligibility for VR protections typically depends on an individual's caste membership. While this information is supposed to be private information, it can often be inferred by the central planner due to various indications such as the individual's last name. A central planner can also obtain this information through documents such as a diploma. Hence, eligibility for VR protections may not be truly private information, and the lack of incentive compatibility of a choice rule may enable a malicious central planner to exploit this information to deny an applicant her open-category HR protections. As documented in Section C.1.2 of the Online Appendix, this type of misconduct not only has been widespread in parts of India, but it even appears to be centrally organized by the local governing bodies in some of its jurisdictions.

3.2. An Easy Fix: 2SMG Choice Rule. Apart from its simplicity, an additional advantage of formulating the SCI-AKG choice rule using its relation to the minimum guarantee choice rule is that, unlike its original formulation that obscures a possible remedy, our equivalent formulation suggests an easy fix. Both anomalies of the SCI-AKG choice rule are caused by the exclusive access given to the general-category individuals for open-category HR protections. This restriction reflects itself in our formulation of the SCI-AKG choice rule during the derivation of the open-category assignments through the formula

$$C^{SCI,o}(I) = C^{o}_{mg}(I^m \cup I^g).$$

Observe that, instead of running the choice rule C^{o}_{mg} for the set of individuals $I^m \cup I^g$, running it for the set of all individuals I provides us with an immediate and intuitive fix. We refer to this alternative mechanism as the two-step minimum guarantee (2SMG) choice rule.

Two-Step Minimum Guarantee (2SMG) Choice Rule $C^{2s}_{mg} = (C^{2s,\nu}_{mg})_{\nu \in \mathcal{V}}$

Given a set of individuals $I \subseteq \mathcal{I}$,

$$C^{2s,o}_{mg}(I) = C^{o}_{mg}(I), \text{ and}$$
$$C^{2s,c}_{mg}(I) = C^{c}_{mg}\big(I^c \setminus C^{o}_{mg}(I)\big) \quad \text{for any } c \in \mathcal{R}.$$

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As one would naturally expect, replacing the SCI-AKG choice rule with the 2SMG choice rule results in a weakly less favorable outcome for members of the general category. The comparison for members of reserve-eligible categories is less straightforward, because in addition to the VR-protected positions, these individuals also compete for the open positions. However, assuming suf cient demand at each reserve-eligible category, replacing the SCI-AKG choice rule with the 2SMG choice rule results in a weakly more favorable outcome in aggregate for members of the reserve-eligible categories.

Proposition 1. For every I *I*

Definition 10. A choice rule $C = (C^\nu)_{\nu \in \mathcal{V}}$ is *non-wasteful* if, for every $I \subseteq \mathcal{I}$, $\nu \in \mathcal{V}$, and $j \in I$,

$$j \notin \widehat{C}(I) \text{ and } |C^\nu(I)| < q^\nu \implies j \notin \mathcal{I}^\nu.$$

That is, if an individual j is declined a position from each one of the categories (thus remaining unmatched) while there is an idle position at some category $\nu \in \mathcal{V}$, then it must be the case that individual j is not eligible for a position at category ν. This mild efficiency axiom has been mandated in India since Indra Sawhney (1992)

Definition 11. A choice rule $C = (C^\nu)_{\nu \in \mathcal{V}}$

this third condition is an implication of another mandate in *Saurav Yadav (2020)*, and therefore this judgment enforces the axiom of *compliance with VR protections* in its stronger form as formulated above.[15]

We are ready to present our first main result, one that has important and previously unknown policy implications for India.

Theorem 1. *Suppose each individual has at most one trait. A choice rule*

1. *maximally accommodates HR protections,*
2. *satisfies no justified envy,*
3. *is non-wasteful, and*
4. *complies with VR protections*

if, and only if, it is the 2SMG choice rule C^{2s}_{mg}.

Prior to its endorsement by the three-judge bench of the Supreme Court in *Saurav Yadav (2020)*, the 2SMG choice rule has been introduced by the justices of the High Court of Gujarat in *Tamannaben Ashokbhai Desai (2020)*, an August 2020 judgment which also mandated the 2SMG choice rule in the state of Gujarat.[16] However, while this choice rule is merely endorsed and not explicitly mandated by *Saurav Yadav (2020)* throughout India, to any possible generalization with overlapping HR protections. For this more general and complex case, central planner announces the number of positions for each category-trait pair,[18] which implicitly implies that they adopt the one-to-one HR matching convention.[19]

4.2. Single-Category Analysis with Overlapping HR Protections. Since HR policies are implemented within vertical categories, we start our analysis with the simple case of a single category. This version of the problem also relates to practical applications other than our main application in India, such as the allocation of K-12 public school seats in Chile where there are overlapping HR protections (Correa et al., 2019).

Throughout Section 4.2, we fix a category $v \in V$.

4.2.1.

position is assigned in Ste[redacted] remaining individual i_2, and therefore the set of selected i[redacted] he second processing sequence of traits.

Example 3 rev[redacted] sequence of traits, the outcome of the minimum [redacted] merit score individuals at the expense of h[redacted] wi[redacted] rence to the horizontal reservation polici[redacted] the me[redacted] f $\{i_1, i_2, i_3\}$ already accommodates th[redacted] is clearly [redacted] a less meritorious group. [redacted] es not only g[redacted] ents of our axioms to account for [redacted] otections, they al[redacted] ritorious horizontal choice rule, intro-[redacted] ion 4.2.3, as a natura[redacted] SMG choice rule.

[redacted]. HR Graph and the Generaliz[redacted] Function. In contrast to the version of [redacted] ur model with non-overlapping HR[redacted] maximizing the accommodation of HR protections is a straightforw[redacted] same for the general version of the model with overlapping HR pro[redacted] nbedding a maximum trait matching procedure within each category[redacted] on the following construction to generalize our HR-maximality funct[redacted] se

(1) to extend our axioms initia[redacted] ction 3 for the model with non-overlapping HR protections,[redacted]
(2) to generalize the 2SMG cho[redacted] l with overlapping HR protections in a way that escapes the sh[redacted] ed in Examples 2 and 3.

[redacted] I^v, construct the following two-[redacted], there are individuals in I. On the [redacted] y v. Let H_t^v denote the set of trait-t [redacted] $_{t \in T}$ H_t^v. There are q_t^v positions in H_t^v [redacted] d a position $p \in H_t^v$ are connected in

De nition 14.

Step 1.k ($k \in \{2, \ldots, \sum_{t \in T} q_t^v\}$): Assuming such an individual exists, choose the highest merit-score individual in $I \setminus I_{k-1}$ who increases the HR utilization of I_{k-1}.[21] Denote this individual by i_k

Two-Step Meritorious Horizontal (2SMH) Choice Rule $C^{2s}_{Ⓜ} = (C^{2s,\nu}_{Ⓜ})_{\nu 2 V}$

For every I *I* ,

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to balance various ethical principles for pandemic medical resource allocation, although their model is not equipped to analyze concurrent implementation of vertical and overlapping horizontal reservation policies.

A few papers study the implementation of vertical or (non-overlapping) horizontal reservations individually in various real-life applications. These include Dur et al. (2018) for school choice in Boston, Dur et al. (2020) for school choice in Chicago, and Pathak et al. (2020a) for H-1B visa allocation in the US. All these models are applications of the more general model in Kominers and S ¨

in matroid theory, we overcome these difficulties with the meritorious horizontal choice rule. More specifically, Proposition 2 and Theorem 2 are conceptually related to abstract results in matroid theory. Proposition 2 can be seen as a generalization of a result in Gale (1968) which shows that the outcome of the Greedy algorithm “dominates” any independent set of a matroid. In our appendix, we refer to this domination relation as “Gale domination.” The first step of our meritorious horizontal choice rule corresponds to the Greedy algorithm defined on an adequately defined matroid, and Proposition 2 shows that this choice rule Gale dominates any choice rule that maximally complies with HR protections. The proof uses mathematical induction on the number of individuals chosen at the second step of our choice rule and uses Gale's result for the base case. Parts of the proof of Theorem 2 uses the properties of the Greedy algorithm.

More broadly, our paper contributes to the field of market design, where economists

(1) all allocation rules for public recruitment are federally mandated to satisfy no justified envy, and thereby
(2) the SCI-AKG choice rule, mandated for 25 years, becomes rescinded.

Using several of the same judgments we present in Section C.1.1 of the Online Appendix, the justices also highlighted the inconsistencies between several high court judgments in relation to desiderata we formulated as the axiom of no justified envy. The justices also declared that while the "first view" that enforces no justified envy by the high court judgments of Rajasthan, Bombay, Gujarat, and Uttarakhand is "correct and rational," the "second view" that allows for justified envy by the high court judgments of Allahabad and Madhya Pradesh is not.[24]

While the axiom of no justified envy becomes federally enforced with Saurav Yadav v State of Uttar Pradesh (2020), unlike in Anil Kumar Gupta (1995) no explicit procedure is mandated with this new Supreme Court ruling. Two points, however, are important to emphasize in this regard. The first one is that prior to Saurav Yadav (2020), the 2SMG choice rule became mandated in the state of Gujarat with the August 2020 high court judgment Tamannaben Ashokbhai Desai v. Shital Amrutlal Nishar (2020).[25] While the justices of the Supreme Court have not enforced any specific rule in their December 2020 judgment, they endorsed the 2SMG choice rule given in Tamannaben Ashokbhai Desai v. Shital Amrutlal Nishar (2020):

> 36. Finally, we must say that the steps indicated by the High Court of Gujarat in para 56 of its judgment in Tamannaben Ashokbhai Desai contemplate the correct and appropriate procedure for considering and giving effect to both vertical and horizontal reservations. The illustration given by us deals with only one possible dimension. There could be multiple such possibilities. Even going by the present illustration, the first female candidate allocated in the vertical column for Scheduled Tribes may have secured higher position than the candidate at Serial No.64. In that event said candidate must be shifted from the category of Scheduled Tribes to Open / General category causing a resultant vacancy in the vertical column of Scheduled Tribes. Such vacancy must then enure to the benefit of the candidate in the Waiting List for Scheduled Tribes - Female.

[24]It is important to emphasize that, prior to this ruling, the second view—now deemed incorrect and irrational—was in line with the SCI-AKG choice rule, whereas the first view—now deemed correct and rational—deviated from the previously mandated choice rule.

[25]The mandated choice rule in Gujarat is described for a single group of beneficiaries (women) for horizontal reservations under this High Court ruling. See Section B.5 in the Online Appendix for the description of the procedure in Tamannaben Ashokbhai Desai (2020).

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Appendix

The restriction of matroid $(E, \mathcal{M})$ to $E' \subseteq E$ is a matroid $(E', \mathcal{M}')$ where $\mathcal{M}' = \{X \subseteq E' : X \in \mathcal{M}\}$. The rank of $X \subseteq E$ is defined as the cardinality of a maximal independent set in the restriction of $(E, \mathcal{M})$ to X. Since all maximal independent sets have the same cardinality, the rank of a set is well-defined. The rank of $X \subseteq E$ is is denoted by $r(X)$. The rank function satisfies the following properties:

R1: If $X \subseteq E$, then $0 \leq r(X) \leq |X|$.
R2: If If $X \subseteq Y \subseteq E$, then $r(X) \leq r(Y)$.
R3: If $X, Y \subseteq E$, then

$$r(X \cup Y) + r(X \cap Y) \leq r(X) + r(Y).$$

A.2. Greedy Choice Rule and Its Properties. For a given weight function $w : E \to \mathbb{R}_+$ that takes distinct values, the greedy algorithm chooses the element with the highest weight subject to the constraint that the chosen set of elements is independent.

Greedy Algorithm

Step 1: Set $X_0 = \emptyset$ and $i = 0$.
Step 2: If there exists $e \in E \setminus X_i$ such that $X_i \cup \{e\} \in \mathcal{M}$, then choose such an element e_{i+1} of maximum weight, let $X_{i+1} = X_i \cup \{e_{i+1}\}$, and go to Step 3; otherwise let $B = X_i$ and go to Step 4.
Step 3: Add 1 to i and go to Step 2.
Step 4: Stop.

The textbook definition of the Greedy algorithm takes w to be any weight function that can take same values for different elements of E. In this case, the Greedy algorithm can select different sets depending on how elements are chosen when they have the same weight. To avoid this issue, we assume that distinct elements of E have different weights.

The greedy algorithm is defined on matroid $(E, \mathcal{M})$. However, it can be applied to any restriction of this matroid. Therefore, the greedy algorithm can be viewed as a single-category choice rule on 2^E (Fleiner, 2001). For the rest of the paper, we view it as a single-category choice rule and refer to it as the greedy choice rule

The greedy algorithm chooses an independent set that has the maximum weight, where the weight of a set is the sum of weights of individual elements. Before we introduce a stronger property of the greedy algorithm, we need the following definition.

Let elements of the sets $X, Y \subseteq E$ be enumerated such that,

$$\text{for every } i, j \in \{1, \ldots, |X|\}, \quad i \leq j \implies w(x_i) \geq w(x_j), \text{ and}$$
$$\text{for every } i, j \in \{1, \ldots, |Y|\}, \quad i \leq j \implies w(y_i) \geq w(y_j).$$

Then, the set $X = \{x_1, \dots, x_{|X|}\} \subseteq E$ Gale dominates the set $Y = \{y_1, \dots, y_{|Y|}\} \subseteq E$ if $|X| \geq |Y|$ and, for every $i \in \{1, \dots, |Y|\}$,

$$w(x_i) \geq w(y_i).$$

We use the notation $X \succeq^G Y$ to denote set X Gale dominates set Y.

The following property of the greedy choice rule is the driving force for a similar property of the meritorious horizontal choice rule that is presented in Proposition 2.

Lemma 1. (Gale, 1968) For every $E' \subseteq E$, the outcome of the greedy choice rule for E' Gale dominates any independent subset of E'.

The following property of choice rules plays an important role in market design.

Definition 16. (Kelso and Crawford, 1982) A choice rule $C : 2^E \to 2^E$ satisfies the *substitutes condition*, if, for every $E' \subseteq E$,

$$e \in C(E') \text{ and } e' \in E' \setminus \{e\} \implies e \in C(E' \setminus \{e'\}).$$

We use the following result in some of our proofs.

Lemma 2.

Lemma 3. A choice rule C: 2^E

the greedy choice rule for the transversal matroid. We use this important observation in proofs of Proposition 2 and Theorem 2 presented below.

Proof of Proposition 1. Let $I \subseteq \mathcal{I}$ be a set of individuals and $I^m \subseteq I$ be the set of reserve-eligible individuals considered at Step 1 of $\widehat{C}^{SCI}$ when I is the set of applicants.

Let $i \in \widehat{C}^{2s}_{mg}(I) \cap I^g$. Then $i \in C^o_{mg}(I) \cap I^g$ because $\widehat{C}^{2s}_{mg}(I) \cap I^g = C^o_{mg}(I) \cap I^g$. Since C^o_{mg} satisfies the substitutes condition (Echenique and Yenmez, 2015), $i \in C^o_{mg}(I^m \cup I^g)$ because $i \in I^g$ and $i \in C^o_{mg}(I)$. Therefore, $i \in C^o_{mg}(I^m \cup I^g) \cap I^g$, which implies $i \in \widehat{C}^{SCI}(I) \cap I^g$ because $\widehat{C}^{SCI}(I) \cap I^g = C^o_{mg}(I^m \cup I^g) \cap I^g$. Therefore, we conclude that $\widehat{C}^{2s}_{mg}(I) \cap I^g \subseteq \widehat{C}^{SCI}(I) \cap I^g$.

The assumption that $|I^c| \geq q^o + q^c$ for each reserve-eligible category $c \in \mathcal{R}$ implies that all category-c positions are filled under both $\widehat{C}^{2s}_{mg}$ and $\widehat{C}^{SCI}$. In addition, the first part of the proposition implies that there are weakly more individuals with reserved categories

$J' = C^{v}_{Ⓜ}(C^{v}(I))$, and $K' = C^{v}(I) \setminus J'$. If $|K'| = 0$, then the proof is complete as in the base case using Lemma 1. For the rest of the proof suppose that $|K'| > 0$.

Lemma 4. There exists $j \in K$ and $j' \in K'$ such that $s(j) \geq s(j')$.

Proof. Suppose, for contradiction, that for every $j \in K$ and $j' \in K'$ we have $s(j) < s(j')$. Since $j \in K = C^{v}_{R}(I)$, $j' \notin K$, and $s(j') > s(j)$, we must have $j' \in$ $I)$

Proof. $C^{\vee}_{Ⓜ}$

Proof of Theorem 3. Let $C = (C^v)_{v \in \mathcal{V}}$ be a choice rule that complies with VR protections, maximally accommodates HR protections, satisfies no justified envy, and is non-wasteful. We show this result using the following lemmas.

Lemma 10. $C^o = C^{2s,o}_{Ⓜ}$.

Proof. We prove that C^o maximally accommodates category-o HR protections, satisfies no justified envy, and is non-wasteful.

First, we show that C^o maximally accommodates category-o HR protections. Suppose, for contradiction, that $n^o(C^o(I)) < n^o(I)$ for some $I \subseteq \mathcal{I}$. Then there exists $i \in I \setminus C^o(I)$ such that $n^o(C^o(I) \cup \{i\}) = n^o(C^o(I)) + 1$. If $i \in I \setminus \hat{C}(I)$, then we get a contradiction with the assumption that C maximally accommodates HR protections. Otherwise, if $i \in C^c(I)$ where $c \in \mathcal{R}$, then we get a contradiction with the assumption that C complies with VR protections. Therefore, C^o maximally accommodates category-o HR protections.

Next, we show that C^o satisfies no justified envy. Let $i \in C^o(I)$ and $j \in I \setminus C^o(I)$ such that $s(j) > s(i)$. If $j \in I \setminus \hat{C}(I)$, then

$$n^o\left((C^o(I) \setminus \{i\}) \cup \{j\}\right) < n^o\left(C^o(I)\right)$$

because C satisfies no justified envy. However, if $i \in C^c(I)$ for category $c \in \mathcal{R}$, then

$$n^o\left((C^o(I) \setminus \{i\}) \cup \{j\}\right) < n^o\left(C^o(I)\right)$$

because C complies with VR protections. Therefore, C^o satisfies no justified envy.

Now, we show that C^o is non-wasteful, which means that $|C^o(I)| = \min\{|I|, q^o\}$ for every $I \subseteq \mathcal{I}$. If there exists an individual $i \in I$ such that $i \notin \hat{C}(I)$, then $|C^o(I)| = q^o$ because C is non-wasteful. If there exists an individual $i \in I$ such that $i \in C^c(I)$ where $c = r(i) \in \mathcal{R}$, then $|C^o(I)| = q^o$ because C complies with VR protections. If these two conditions do not hold, then all the individuals are allocated open-category positions, i.e., $I = C^o(I)$. Therefore, under all possibilities, we get $|C^o(I)| = \min\{|I|, q^o\}$, which means that C^o is non-wasteful.

Since C^o maximally accommodates category-o HR protections, satisfies no justified envy, and is non-wasteful, we get $C^o = C^o_{Ⓜ}$ (Theorem 2), and hence $C^o = C^{2s,o}_{Ⓜ}$.

Let $c \in \mathcal{R}$, $I \subseteq \mathcal{I}$, and $\overline{I^c} = \{i \in I \setminus C^o_{Ⓜ}(I) | r(i) = c\}$.

Lemma 11. $C^c(I)$ maximally accommodates category-c HR protections for $\overline{I^c}$.

Proof. Suppose, for contradiction, that $n^c(C^c(I)) < n^c(\overline{I^c})$. This is equivalent to

$$n^c(C^c(I)) < n^c(\overline{I^c}) = n^c\left(C^c(I) \cup \{i \in I \setminus \hat{C}(I) | r(i) = c\}\right),$$

which implies that there exists $i \in I \setminus \widehat{C}(I)$ who is eligible for category c such that

$$n^c(C^c(I \cup \{i\})) = n^c(C^c(I)) + 1.$$

This equation contradicts the assumption that C maximally accommodates HR protections. Therefore, $C^c(I)$ maximally accommodates category-c HR protections for $\bar{I}^c$.

Lemma 12. $C^c(I)$ satisfies no justified envy for $\bar{I}^c$.

Proof. Let $i \in C^c(I)$ and $j \in \bar{I}^c \setminus C^c(\bar{I}^c)$ be such that $s(j) > s(i)$. Note that $i \in \bar{I}^c$. Since C satisfies no justified envy, we have

$$n^c(C^c(I)) > n^c((C^c(I) \setminus \{j\}) \cup \{i\}).$$

Hence, C^c satisfies no justified envy for $\bar{I}^c$.

Lemma 13. $|C^c(I)| = \min\{|\bar{I}^c|, q^c\}$.

Proof. We consider two cases. First, if $C^c(I) = \bar{I}^c$, then $|C^c(I)| = \min\{|\bar{I}^c|, q^c\}$ because $|C^c(I)| \le q^c$. Otherwise, if $C^c(I) \neq \bar{I}^c$, then there exists $i \in \bar{I}^c \setminus C^c(I)$. Therefore, $i \in I \setminus \widehat{C}(I)$. Since C is non-wasteful, we get $|C^c(I)| = q^c$ Since $i \in \bar{I}^c \setminus C^c(I)$ and $|C^c(I)| = q^c$, $|\bar{I}^c| > q^c$. Therefore, $|C^c(I)| = q^c = \min\{|\bar{I}^c|, q^c\}$.

Therefore, $C^c(I)$ maximally accommodates category-c HR protections for $\bar{I}^c$, $C^c(I)$ satisfies no justified envy for $\bar{I}^c$, and $C^c(I)$ is non-wasteful for $\bar{I}^c$. By Theorem 2, $C^c(I) = C^c_{\textcircled{M}}(\bar{I}^c)$ and, thus,

$$C^c(I) = C^c_{\textcircled{M}}(\bar{I}^c) = C^c_{\textcircled{M}}(\{i \in I \setminus C^o_{\textcircled{M}}(I) \mid r(i) = c\}) = C^{2s,c}_{\textcircled{M}}(I).$$

Proof of Proposition 3. Suppose that i is chosen by $\widehat{C}^{2s}_{\textcircled{M}}$ when she withholds some of her reserve-eligible privileges. If i is chosen by $C^o_{\textcircled{M}}$ for an open-category position, then i will still be chosen by declaring all her reserve-eligible privileges because $C^o_{\textcircled{M}}$ does not

Online Appendix

Appendix B. Institutional Background on Vertical and Horizontal Reservations

In this section of the Online Appendix, we present:

(1) the description of the concepts of vertical reservation and horizontal reservation as they are quoted in the Supreme Court judgments *Indra Sawhney (1992)* and *Rajesh Kumar Daria (2007)* in Sections B.1 and B.2,

(2) the main quotes from the Supreme Court judgments *Anil Kumar Gupta (1995)* and *Rajesh Kumar Daria (2007)* that allow us to formulate the SCI-AKG choice rule in Section B.3,

(3) the revised mandates of the Supreme Court judgment *Saurav Yadav (2020)*, which imply the 2SMG choice rule is the only mechanism that remains lawful for the case of non-overlapping horizontal reservations in Section B.4, and

(4) the description of the 2SMG choice rule that is mandated in the State of Gujarat as it is quoted in the August 2020 High Court of Gujarat judgment *Tamannaben Ashokbhai Desai (2020)* in Section B.5.

B.1. Indra Sawhney (1992): Introduction of Vertical and Horizontal Reservations. The terms vertical reservation and horizontal reservation are coined by the Constitution bench of the Supreme Court of India, in the historical judgment *Indra Sawhney (1992)*, where

- the former was formulated as a policy tool to accommodate the higher-level protective provisions sanctioned by Article 16(4) of the Constitution of India, and
- the latter was formulated as a policy tool to accommodate the lower-level protective provisions sanctioned by Article 16(1) of the Constitution of India.

The description of these two affirmative action policies and how they are intended to interact with each other is given in the judgment with following quote:

> A little clarification is in order at this juncture: all reservations are not of the same nature. There are two types of reservations, which may, for the sake of convenience, be referred to as 'vertical reservations' and 'horizontal reservations'. The reservation in favour of scheduled castes, scheduled tribes and other backward classes [under Article 16(4)] may be called vertical reservations whereas reservations in favour of physically handicapped [under clause (1) of Article 16] can be referred to as horizontal reservations. Horizontal reservations cut across the vertical reservations -- what is called interlocking reservations. To be more precise, suppose 3% of the vacancies are reserved in favour of physically handicapped persons; this would be a reservation relatable to

clause (1) of Article 16. The persons selected against his quota will be placed in the appropriate category; if he belongs to SC category he will be placed in that quota by making necessary adjustments; similarly, if

> to vertical (social) reservations will not apply to horizontal (special) reservations. Where a special reservation for women is provided within the social reservation for Scheduled Castes, the proper procedure is first to fill up the quota for scheduled castes in order of merit and then find out the number of candidates among them who belong to the special reservation group of 'Scheduled Castes-Women'. If the number of women in such list is equal to or more than the number of special reservation quota, then there is no need for further selection towards the special reservation quota. Only if there is any shortfall, the requisite number of scheduled caste women shall have to be taken by deleting the corresponding number of candidates from the bottom of the list relating to Scheduled Castes. To this extent, horizontal (special) reservation differs from vertical (social) reservation. Thus women selected on merit within the vertical reservation quota will be counted against the horizontal reservation for women.

B.3. Anil Kumar Gupta (1995): Implementation of Horizontal Reservations Compartmentalized within Vertical Reservations. While horizontal reservations can be implemented either as overall horizontal reservations for the entire set of positions, or as compartment-wise horizontal reservations within each vertical category including the open category (OC), the Supreme Court recommended the latter in their judgment of Anil Kumar Gupta (1995):

> We are of the opinion that in the interest of avoiding any complications and intractable problems, it would be better that in future the horizontal reservations are comparmentalised in the sense explained above. In other words, the notification inviting applications should itself state not only the percentage of horizontal reservation(s) but should also specify the number of seats reserved for them in each of the social reservation categories, viz., S.T., S.C., O.B.C. and O.C.

The procedure to implement compartmentalized horizontal reservation is described in Anil Kumar Gupta (1995) as follows:

> The proper and correct course is to first fill up the O.C. quota (50%) on the basis of merit: then fill up each of the social reservation quotas, i.e., S.C., S.T. and B.C; the third step would be to find out how many candidates belonging to special reservations have been selected on the above basis. If the quota fixed for horizontal reservations is already satisfied - in case it is an over-all horizontal reservation - no further question arises. But if it is not so satisfied, the

requisite number of special reservation candidates shall have to be taken

> illustration given by us deals with only one possible dimension. There could be multiple such possibilities. Even going by the present illustration, the first female candidate allocated in the vertical column for Scheduled Tribes may have secured higher position than the candidate at Serial No.64. In that event said candidate must be shifted from the category of Scheduled Tribes to Open / General category causing a resultant vacancy in the vertical column of Scheduled Tribes. Such vacancy must then enure to the benefit of the candidate in the Waiting List for Scheduled Tribes -- Female.

More speci cally the quote formulates the mandate that a member of a reserve-eligible category (Scheduled Tribes in the example) has to be considered for open-category HR-protected positions (for women HR protections in the example) before using up a VR-protected position. Apart from its enforcement of our axiom compliance with VR protections, ths9dlg23(quote)5025(aso9dlg23brsins9dlg23clarity9dlg23(for)-521(the)-523(fallwting)-523

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choice rule is given in the judgment only for a single horizontal trait (women), it is also well-defined and well-behaved for multiple (but non-overlapping) traits as presented in Theorem 1. Originally introduced in Sönmez and Yenmez (2019) prior to the judgment of the High Court of Gujarat in December 2020, the 2SMG choice rule is endorsed by the Supreme Court judgment Saurav Yadav (2020) for the entire country.[26] Paragraph 56 of the High Court of Gujarat judgment Tamannaben Ashokbhai Desai (2020) describes the mandated procedure as follows:

> For the future guidance of the State Government, we would like to explain the proper and correct method of implementing horizontal reservation for women in a more lucid manner.

up for the shortfall of women.

Step 5: Do a check for horizontal reservation in the Step 4 List of SCs.

If there are 4 SC women, the quota of 33% is complete. Nothing more is to be done. If there is a shortfall of SC women (say, only 2 women are available), 2 more women have to be added. The way to do this is to, first, delete the last 2 male SC candidates of the Step 4 List and then to go down the Step 1 List after item no. 51, and pick the first 2 SC women. As soon as 2 such SC women in Step 1 List are found, they are to be brought up and added to the Step 4 List of SCs to make up for the shortfall of SC women. Now, the 33% quota for SC women is fulfilled. List of SCs is to be locked. Step 4 List becomes final. If 2 SC women cannot be found till the last number in the Step 1 List, these 2 vacancies are to be filled up by SC men. If in case, SC men are also wanting, the social reservation quota of SC is to be carried forward to the next recruitment unless there is a rule which permits conversion of SC quota to OC.

Step 6: Repeat steps 4 and 5 for preparing list of STs.

Step 7: Repeat steps 4 and 5 for preparing list of SEBCs.

Appendix C. Documentation of Evidence from Indian Court Rulings on Disruption Caused by the Flaws of the SCI-AKG Choice Rule

In this section we present extensive evidence on the disarray caused by the shortcomings of the SCI-AKG choice rule in India. Much of our analysis, the High Court judgments presented Section C.1.1, and our policy recommendations parallel the arguments and the decision of the December 2020 Supreme Court judgment Saurav Yadav v State of Uttar Pradesh (2020)Our entire analysis and policy recommendations predate this important judgment, and it was already presented in an earlier draft of this paper in S önmez and Yenmez (2019).

C.1. Litigations on the SCI-AKG Choice Rule. As we have argued in Section 3.1, the SCI-AKG choice rule fails our axioms of no justi ed envy. Moreover, it also fails incentive compatibilitydue to backward class candidates losing their open-category HR protections upon claiming their VR protections by declaring their backward class status.

The failure of SCI-AKG choice rule to satisfy no justi ed envy is fairly straightforward to observe. All it takes is a rejected backward class candidate to realize that her merit score is higher than an accepted general-category candidate, even though she quali es for all the HR protections the less-deserving (but still accepted) candidate quali es for. Since the primary role of the reservation policy is positive discrimination for candidates with more vulnerable backgrounds, this situation is very counterintuitive, and it often results in legal action. Focusing on complications caused by either anomaly, we next present several court cases to document how they handicap concurrent implementation of vertical and horizontal reservation policies in India.

C.1.1. *High Court Cases Related to Justified Envy*.

> seats. The positions applied for included that of teachers Grade-II and III, school lecturers, headmasters and pharmacists.

Ironically, while the High Court's decision is correct, it also means that the better-behaved version of the choice rule has to be abandoned by the state.

(2) *Ashish Kumar Pandey And 24 Others vs State Of U.P. And 29 Others on 16 March, 2016*, Allahabad High Court .[29] This lawsuit was brought to Allahabad High Court by 25 petitioners, disputing the mechanism employed by the State of Uttar Pradesh—the most populous state in India with more than 200 million residents—to apply the provisions of horizontal reservations in their allocation of more than 4,000 civil police and platoon commander positions. Of these positions, 27%, 21%, and 2% are each vertically reserved for members of Other Backward Classes (OBC), Scheduled Castes (SC), and Scheduled Tribes (ST), respectively, and 20%, 5%, and 2% are each horizontally reserved for women, ex-servicemen, and dependents of freedom ghters, respectively. While only 19 women are selected for open-category positions based on their merit scores, the total number of female candidates is less than even the number of open-category horizontally reserved positions for women, and as such all remaining women are selected. However, instead of assigning them positions from their respective backward class categories (as it is mandated under the SCI-AKG choice rule), all of them are assigned positions from the open category. Similarly, backward class candidates are deemed eligible to use horizontal reservations for dependents of freedom ghters and ex-servicemen as well. The counsel for the petitioners argues that not only did the State of U.P. make an error in its implementation of horizontal reservations, but also that the error was intentional. The following quote is from the court case:

> Per contra, learned counsel appearing for the petitioners would submit that fallacy was committed by the Board deliberately, and with malafide intention to deprive the meritorious candidates their rightful placement in the open category. The candidates seeking horizontal reservations belonging to OBC and SC category were wrongly adjusted in the open category, whereas, they ought to have been adjusted in their quota provided in respective social category. The action of the Board is not only motivated, but purports to take forward the unwritten agenda of the State Government to accommodate as many number of OBC/SC candidates in the open category.

[29]The case is available at`https://indiankanoon.org/doc/74817661/` (last accessed on 03/07/2019).

The judge sides with the petitioners, and rules that the State of Uttar Pradesh must correct its erroneous application of the provisions of horizontal reservations. The judge further emphasizes that the State has played foul, stating:

> There is merit in the submission of the learned counsel for the petitioners that the conduct of the members of the Board appears not only mischievous but motivated to achieve a calculated agenda by deliberately keeping meritorious candidates out of the select

> reservation and horizontal reservation and the way and manner in which the provision has to be pressed and brought into play.

(3) *Asha Ramnath Gholap vs President, District Selection Committee & Ors. on March 3rd, 2016*, Bombay High Court .[31] In this case, there are 23 pharmacist positions to be allocated; 13 of these positions are vertically reserved for backward classes and the remaining 10 are open for all candidates. In the open category, 8 of the 10 positions are horizontally reserved for various groups, including 3 for women. The petitioner, Asha Ramnath Gholap, is an SC woman, and while there is one vertically reserved position for SC candidates, there is no horizontally reserved position for SC women. Under the SCI-AKG choice rule, she is not eligible for any of the horizontally reserved positions for women at the open category. Nevertheless, she brings her case to the Bombay High Court based on an instance of justi ed envy, described in the court records as follows:

> It is the contention of the petitioner that Respondent Nos. 4 & 5 have received less marks than the petitioner and as such, both were not liable to be selected. The petitioner has, therefore, approached this court by invoking the writ jurisdiction of this court under Article 226 of the Constitution of India, seeking quashment of the select list to the extent it contains the names of Respondent Nos.4 and 5 against the seats reserved for the candidates belonging to open female category.

Under the federal law, there is no merit to this argument, because the SCI-AKG choice rule allows for justi ed envy. However, the judges side with the petitioner on the basis that a candidate cannot be denied a position from the open category based on her backward class membership, essentially ruling out the possibility of justi ed envy under a Supreme Court-mandated choice rule, which is designed to allow for positive discrimination for vulnerable groups. [32] Their justi cation is given in the court records as follows:

[31]The case is available at `https://indiankanoon.org/doc/178693513/` (last accessed on 03/08/2019).

[32]In a very similar Bombay High Court case Rajani Shaileshkumar Khobragade ... vs The State Of Maharashtra And ... on 31 March, 2017where the petitioner led a lawsuit based on another instance of justi ed envy, the judges of the same high court dismissed the petition. This case is available at `https://indiankanoon.org/doc/7250640/`, last accessed on 03/09/2019. Indeed, there seem to be several con icting decisions at the Bombay High Court on this very issue, including a series of cases reported in a The Times of India story dated 07/18/2018 "MPSC won't issue job letters till HC hears plea on quota issue" available at `https://timesofindia.indiatimes.com/city/aurangabad/mpsc-wont-issue-job-letters-till-hc-hears-plea-on-quota-issue/articleshow/65029505.cms` (last accessed on 03/09/2019).

> We find the argument advanced as above to be fallacious. Once it is held that general category or open category takes in its sweep all candidates belonging to all categories irrespective of their caste, class or community or tribe, it is irrelevant whether the reservation provided is vertical or horizontal. There cannot be two interpretations of the words 'open category' ...

(4) *Uday Sisode vs Home Department (Police) on 24 October, 2017*, Madhya Pradesh High Court .[33] In another case parallel to that at the Bombay High Court, the judges of the Madhya Pradesh High Court issued a questionable decision by siding with a petitioner who led this lawsuit based on another instance of justi ed envy.

(5) *Smt. Megha Shetty vs State Of Raj. & Anr on 26 July, 2013*, Rajasthan High Court .[34] In contrast to Asha Ramnath Gholap (2016) and Uday Sisode (2017) where the judges have been erroneous siding with petitioners whose lawsuits are based on instances of justi ed envy, in this case a general category petitioner seeks legal action against the state on the basis that several HR-protected open-category positions for women are allocated to women from OBC who are not eligible for these positions (unless they receive it without invoking the bene ts of horizontal reservation). While all these OBC women have higher merit scores than the petitioner and the state has apparently used a better behaved procedure, the petitioner's case has merit because the SCI-AKG choice rule allows for justi ed envy in those situations. In an earlier lawsuit, the petitioner's case was already declined by a single judge of the same court based on an erroneous interpretation of Indra Sawhney (1992)

As seen from this argument, many judges have dif culty perceiving that the

> State of Maharashtra dated 26.07.2017, whereunder it is prescribed that a female candidate belonging to any reserved category, even if tenders application form seeking employment as an open category candidate, the name of such candidate shall not be recommended for employment against a open category seat.

Moreover, not all decisions in these lawsuits are made in accordance with the SCI-AKR choice rule, which allows candidates to forego their VR (or HR) protections. This is the case both for the first and last lawsuit listed above. For example, in the last lawsuit, two petitioners each applied for a position without declaring their backward class membership, with the intention to benefit from open-category HR protections. Following their application, these petitioners were requested to provide their school leaving certificates, which provided information on their backward class status. Upon receiving this information, the petitioners were declined eligibility for open-category HR protections, even though they never claimed their VR protections. Hence, they filed the fourth lawsuit given above. Remarkably, their petition was declined on the basis of their backward class membership. Here we have a case where the authorities not only go to great lengths to obtain the backward class membership of the candidates, and wrongfully decline their eligibility for open category HR protections, but they also manage to get their lawsuits dismissed. The mishandling of this case is consistent with the concerns indicated in the February 2006 issue of *The Inter-Regional Inequality Facility* policy brief:[38]

> Another issue relates to the access of SCs and STs to the institutions of justice in seeking protection against discrimination. Studies indicate that SCs and STs are generally faced with insurmountable obstacles in their efforts to seek justice in the event of discrimination. The official statistics and primary survey data bring out this character of justice institutions. The data on Civil Rights cases, for example, shows that only 1.6% of the total cases registered in 1991 were convicted, and that this had fallen to 0.9% in 2000.

C.1.3. *Loss of Access to HR protections without any Access to VR protections.* The main justification offered in various Supreme Court cases for denying backward class members their open-category HR protections is avoiding a situation where an excessive number of positions are reserved for members of these classes. In several cases, however, members of these classes are denied access to open-category HR protections even when the number

[38]The policy brief is available at `https://www.odi.org/sites/odi.org.uk/files/odi-assets/publications-opinion-files/4080.pdf` (last accessed 03/09/2019).

of VR-protected positions is zero for their reserve-eligible vertical category. This is the case in the following two lawsuits:

(1) Tejaswini Raghunath Galande v. The Chairman, Maharashtra Public Service Commission and Ors. on 23 January 2019, Writ Petition Nos. 5397 of 2016 & 5396 of 2016, High Court of Judicature at Bombay.[39]
(2) Original Application No. 662/2016 dated 05.12.2017, Maharashtra Administrative Tribunal, Mumbai. [40]

In both cases, while the petitioners claimed their VR protections, there was no VR-protected position for their class. Yet in both cases petitioners lost their open-category HR protections. In the first case, the petitioners' lawsuit to benefit from open-category HR protections was initially declined by a lower court, resulting in an appeal at the High Court. The lower court's decision was overruled in the High Court, and her request was granted. On the other hand, the second petitioner's similar request was declined by the Maharashtra Administrative Tribunal. What is more worrisome in the second case is that, while initially three positions were VR-protected for the petitioner's backward class, after the petitioner's application these VR-protected positions were withdrawn. Therefore, the candidate declared her backward class status, giving up her open-category HR protection, presumably to gain access to VR-protected positions set aside for her reserve-eligible class, only to learn that she had given up her eligibility for nothing.

Appendix D. Original Formulation of the AKG-SCI Choice Rule and Its Equivalence to Our Formulation

The mechanics for implementing HR protections is described in the two Supreme Court judgments Anil Kumar Gupta (1995) and Rajesh Kumar Daria (2007), and given in Section B.3. In the main body of the paper we used a simpler formulation of the SCI-AKG choice rule, that relies on its relation to the minimum guarantee choice rule. In this section of the Online Appendix we formulate the original description of the SCI-AKG choice rule and prove its equivalence to our simpler formulation.

Both judgments describe the procedure for a single trait, although the procedure can be repeated sequentially for each trait. In our description below, we adhere to this straightforward extension of the procedure.

candidates (across all categories) first, followed by the positions at each reserve-eligible category to the highest merit score remaining candidates from these categories. This is indeed the first step of the SCI-AKG choice rule. Once a tentative assignment is determined, the necessary adjustments are subsequently made to implement HR protections, first for the open-category positions, then for positions at each reserve-eligible category. The adjustment process is repeated for each trait.

Formally, for a given category $v \in V$ of positions, let a set of individuals $J \subseteq I^v$ who are tentatively assigned to category-v positions and a set of individuals $K \subseteq I^v \setminus J$ who are eligible for horizontal adjustments at category v be such that

(1) $|J| = q^v$ and
(2) $s(i) > s(i')$ for any $i \in J$ and $i' \in K$.

Then, for a given processing sequence $t^1, t^2, \ldots, t^{|T|}$ of traits, the horizontal adjustment process is carried out with the following procedure.

AKG Horizontal Adjustment Subroutine (AKG-HAS)

Step 1 (Trait-t^1 adjustments): Let r_1 be the number of individuals in J with trait t^1.

Case 1. $r_1 \geq q^v_{t^1}$

Let J^1 be the set of $q^v_{t^1}$ individuals with the highest merit scores in J with trait t^1. Finalize their assignments as the recipients of trait-t^1 HR-protected positions within category v. Proceed to Step 2.

Case 2. $r_1 < q^v_{t^1}$

Let J^1_m be the set of all individuals in J with trait t^1. Let s_1 be the number of individuals in K who have trait t^1. Let J^1_h be

Let J^k_m be the set of all individuals in $J \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ with trait t^k. Let s_k be the number of individuals in $K \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ with trait t^k. Let J^k_h be

- the set of $(q^v_{t^k} - |J^k_m|)$ individuals with the highest merit scores in $K \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ who have trait t^k if $s_k \geq q^v_{t^k} - |J^k_m|$, and
- the set of all individuals in $K \setminus \bigcup_{\ell=1}^{k-1} J^\ell$ who have trait t^k if $s_k < q^v_{t^k} - |J^k_m|$.

Let $J^k = J^k_m \cup J^k_h$ and finalize their assignments as the recipients of trait-t^k HR-protected positions within category v. Proceed to Step $k+1$.

Step $(|T|+1)$ (Finalization of category-v no-trait assignments) : Let J^0 be the set of $q^v - \sum_{\ell=1}^{|T|} |J^\ell|$ individuals with the highest merit scores in $J \setminus \bigcup_{\ell=1}^{|T|} J^\ell$

The procedure selects the set of individuals in $\bigcup_{\ell=}^{|T|}$

Proof of (1): $|I'| = q^v$ follows because at Step $|T| + 1$ of AKG-HAS all positions are lled.

Proof of (2): Let $i \in I'$ and $j \in I \setminus I'$ such that $s(j) > s(i)$. Since $j \notin I'$, either j does not have a trait or there are at least q^v_t individuals in I' where t is j's only trait. If j does not have a trait, then i must have a trait t' such that the number of individuals in I' who have trait t' is $\min\{q^v_{t'}, |\{i' \in I : t' \in t(i')\}|\}$. Then $n^v((I'$

If $jI^c\, n\, C^{SCI,o}(I)j$ q^c then assign all individuals in $I^c\, n\, C^{SCI,o}(I)$ to category-c positions, nalizing the assignments of individuals in I^c. In this case $C^{SCI,c}(I) = I^c\, n\, C^{SCI,o}(I)$.

Otherwise, if $jI^c\, n\, C^{SCI,o}(I)j > q^c$, then tentatively assign the highest merit-score q^c individuals in $I^c\, n\, C^{SCI,o}(I)$ to category-c positions. Let J^c denote the set of individuals who are tentatively assigned to category- c positions in this case.

Step 4 (Finalization of reserve-eligible category positions) : For any reserve-eligible category c 2 R, the set of individuals eligible for category- c horizontal adjustments is $I^c\, n\, (C^{SCI,o}(I)\ [\ J^c)$. For any reserve-eligible category c 2 R, apply the AKG-HAS

to the set J^c of tentative recipients of category- c positions

with the set of individuals in $I^c\, n\, (C^{SCI,o}(I)\ [\ J^c)$ who are eligible for category- c horizontal adjustments

to nalize the set of recipients $C^{SCI,c}(I)$ of category-c positions.

The outcome