Silent Promotion of Agendas

Campaign Contributions and Ideological Polarization

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Abstract

Until recently, both Republican and Democratic administrations have been promoting free trade and market deregulation for decades without intensive policy debates. We set up a two-party electoral competition model in a two-dimensional policy space with campaign contributions by an interest group that promotes a certain agenda. Assuming

1 Introduction

The ideological distance between congressional Democrats and Republicans has risen substantially in the last few decades (McCarty et al. 2016). DW-Nominate scores by Poole and Rosenthal (1985, 1991) show that the voting gap between congressional Democrats and Republicans is now larger than any point in the history.[1] This rise coincided with globalization, market deregulation, rising income inequality, and an increase in campaign spendings and contributions in electoral politics.

These trends interact with each other. It is natural to assume that globalization trend has been a ecting market deregulation, and it is widely acknowledged that globalization and market deregulation have contributed to growing income inequality in the US as well as other countries. However, the mechanism by which globalization and market deregulation can cause policy polarization has not been discussed frequently (a notable exception is Autor et al., 2016).[2] In this paper, we propose a simple and tractable model with multidimensional policy space to explain these interactions.

Historically, both Republican and Democratic administrations have been promoting free trade and (more recently) nancial market deregulation for decades, and there have been few serious debates on the pros and cons between their presidential candidates. Exporting rms have been lobbying for trade liberalization (Kim 2017),[3] and such policies have been promoted by US administrations irrespective of party. Many citizens feared NAFTA (North

Although Democrats have been traditionally the primary opponent of -nancial deregulation, partisan convergence on this issue occurred from the 1980s until the Lehmann shock. The major deregulation was the removal of the interstate branching prohibitions in banking industry, the Riegle-Neal Interstate Banking and Branching E ciency Act of 1994, which was introduced by Democrats and signed into a law by Bill Clinton. Keller and Kelly (2015) nd this convergence since 1980s empirically and show that campaign nance played a role in this partisan convergence.

In order to analyze the relationship between ideological polarization and convergence in free trade/market deregulation issues, we will set up a two-candidate electoral competition model over two-dimensional policies: an ideological dimension and an \agenda"dimension in which both presidential candidates and voters have a bliss point in the policy space. Another key player is an Interest Group (IG) such as a group of exporting rms (in free trade policies) and the Wall Street (in nancial market deregulations): they are interested in promoting the agenda while they do not care about ideological dimension. Voters are assumed to be impressionable, and IG can provide campaign contributions to candidates who would e ectively enhance their likeability by spending money on political advertisements.[6] If both party candidates receive campaign contributions, the risk of electoral competition endangering the promotion of the agenda is removed. We will explore the relationship between IG's promotion of the agenda, the rise of campaign contributions, and political polarization.

We introduce a simple and tractable probabilistic voting model with uncertain valences, in which two party candidates have both o ce and policy motivations. Although it is well-known that majority voting rule is ill-behaved if the policy space is multi-dimensional, we assure the existence of a median voter in our model by adopting a variation of strong assumptions used in

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the key issue in the election. We are not talking about a situation where the candidates are purposely leaving their positions ambiguous unlike in Alesina and Cukierman (1990), Glazer (1990), and Berliant and Konishi (2005). Appendix B illustrates how the 2016 presidential election race was di erent from previous presidential elections.

[6]Campaign contributions include individual contributions and PAC (Political Action Committees) contributions. Barber and McCarty (2015), and McCarty et al. (2016) report that the share thrt

Davis, deGroot, and Hinch (1972).[7] We first establish the existence of electoral equilibrium when there is a median voter (Proposition 1). Then, we assure the existence of the median voter in our model (Proposition 2). After establishing that candidates' incentive compatibility constraints are binding (Proposition 3), we show that candidates' ideological positions polarize and campaign contributions rise analytically (symmetric candidate case: Proposition 5) and numerically (asymmetric cases). The mechanism behind this result is simple: as IG promotes an agenda more than the candidates want, the candidates' payoffs from winning go down. To compensate these losses, candidates choose policies closer to their ideal positions, causing an ideological polarization.[8] This result is not limited to symmetric case. We conduct numerical analysis for asymmetric candidate cases. Our results suggest that, if two candidates are asymmetric in their ideal positions in the agenda dimension, their ideological polarization is also asymmetric as IG promotes the agenda more — that is, the candidate who is less eager to promote the agenda tends to receive more contributions and polarizes her ideological policy more.

The rest of Section 1 reviews related literature. We introduce the model in Section 2. In Section 3, we analyze properties of equilibrium in the electoral competition and incentive compatibility constraints. In Section 4, we provide analytical results when two candidates are symmetric. We discuss the optimal IG contract under different circumstances via numerical analysis in Section 5. In Section 6, we check the robustness of our model by dropping our simplifying assumptions: we will discuss Wittman's candidate payoff function and expected payoff maximization by a moderately risk-averse IG. Section 7 concludes. All proofs are collected in Appendix C.

1.1 Related Literature

Our framework is built on an influential electoral competition model with interest groups by Grossman and Helpman (1996), but there are a number of differences. Following Baron (1994), Grossman and Helpman (1996) assume that there are informed and uninformed voters, and that uninformed voters'

[7] Krasa and Polborn (2010 and 2014) deal with two-dimensional policy space by assuming that candidates are not flexible in choosing their positions on one dimension: e.g., candidates

voting behaviors are affected by campaign contributions (*impressionable voters*). Although Grossman and Helpman (1996) allow general policy space with multiple lobbies, our model restricts the attention to a special policy space with two dimensions | (a) an agenda dimension in which an Interest Group wants to promote, and (b) a standard Hotelling-type ideological dimension. Grossman and Helpman (1996) assume that lobbies influence the parties' policy platforms through contribution functions, while we simply use take-it-or-leave-it offers instead. They analyze one lobby case extensively, and show that the lobby contributes more to a candidate who has a better chance to win, though it makes contributions to both candidates.[9] We also focus on one IG case, and explore the shapes of incentive compatible constraints and the interaction of policies both analytically and numerically.

In the voting stage, we need to use a two-dimensional policy space. It is hard to assure the existence of simple majority voting equilibrium for multiple dimensional policy spaces, even with probabilistic voting (Wittman 1983, Lindbeck and Weibull 1987, Roemer 2001, and Krasa and Polborn 2012).[10] Although we need to adopt a simplifying assumption (\symmetry" in voter distribution), we manage to establish a tractable probabilistic voting model with both office- and policy-motivated candidates, applying the result in Davis et al. (1972). Note, however, that candidates choose different policies in our model, although policy-convergence occurs in Davis et al. (1972). Besides the dimensionality issue, Roemer (1997) proves the existence of pure strategy Nash equilibrium in a setup where the candidates do not have complete information about median voter's bliss point.[11] In contrast, we assume that the uncertainty comes from an additive valence shock following Londregan and Romer (1993).

There is a large body of literature about campaign spending which can be roughly divided into two approaches. The first one assumes that the contribution \impresses" voters directly. In addition to Grossman and Helpman (1996), an incomplete list of this branch includes Meirowitz (2008), Ashworth and Bueno de Mesquita (2009), and Pastine and Pastine (2012). Within this

[9] Grossman and Helpman (1996) analyze the multi-lobby case by applying the insights developed in the single-lobby case.

[10] Krasa and Polborn (2014) provide an interesting electoral competition model in which the Democrat is better at providing public goods than the Republican, and show that income redistribution is discouraged as the Republican party's ideological position polarizes. Greco (2016) presents a model that discourages income redistribution when high-income earners care about ideology more than low-income earners, and provides empirical evidence.

[11] In a similar setup, Bernhardt et al. (2009) provide a sufficient condition for the existence of symmetric equilibrium. See Duggan and Martinez (2017).

branch, our paper is most closely related to Rivas's (2017) model in which two ideologically-motivated interest groups contribute money to o ce-motivated candidates in order to promote extreme policies. He shows that if one lobbying group has a higher valuation then the candidate supported by the lobby polarizes policy more than the other candidate.[12] Chamon and Kaplan (2013) consider an interest group that makes o ers to both candidates with a threat to contribute to the other candidate if the o er is rejected. With this o -equilibrium threat to contribute to the other candidate, the interest group is able to promote its special-interest agenda by controling o ce-motivated candidates without o ering large amount of contributions.[13]

The second approach considers *informative* campaign spending. For example, Austen-Smith (1987) considers contributions as advertising e orts that can reduce uncertainty when voters observe candidates' proposed policies. Prat (2002a and 2002b) models contributions as a signal of unobservable candidate valences. Coate (2004) considers campaign spending as an informative advertisement abocandidatIJidat0020011(prop)-27(ositions.)]TJ/F45 17.2154 Tf 0 -39.933 Td [(2)-1
paign contributions. Players involved are an Interest Group (IG), two party candidates *j 2 fL; Rg*

where $\lambda > 0$ describes the relative importance of the agenda dimension for voters. Note that v is increasing in C (voters are impressionable). The distribution of voters is described by the distribution of voters' bliss points. Voters' bliss points are distributed with density function $f : P \times A \to \mathbb{R}_+$ on policy space $P \times A$.

There is an Interest Group (IG) that cares about agenda dimension $a \in A$. To simplify the analysis, we assume that IG intends to achieve $\tilde{a}$ no matter who wins, and that IG tries to spend as little as possible to achieve $\tilde{a}$ through the election process using its contributions to two candidates.[14] Therefore, we can simplify its offer as $(C_L; C_R)$. IG proposes $(C_L; C_R)$, and the political contribution C_j is contingent on candidate j's commitment to adopting policy $\tilde{a}$ (C_j will be spent as campaign expenses in the election). Candidate j needs to decide whether to take IG's offer C_j or not. If candidate j chooses *not* to take the offer, she can choose p_j and a_j freely, but needs to run her campaign without IG's contributions. In this case, we set her campaign spending to $C_j = 0$. On the other hand, if she chooses to take the offer, she can only compete with the p_j (since she has committed to $a_j = \tilde{a}$), but with C_j as her covered campaign expenses.[15] Once a candidate annouce her policies, she must commit to them.

We assume that there is uncertainty in election outcomes due to random valence terms for the candidates, which are common to all voters (Wittman 1983). The valence vector $\epsilon = (\epsilon_L; \epsilon_R)$ is composed of two random variables such that voter $(p; a)$ evaluates L and R by[16]

$$v_{(p;a)}(p_L; a_L; C_L$$

that our result is qualitative robust even without this assumption.[17] The equilibrium concept adopted is the subgame perfect Nash equilibrium (SPNE). We solve the political game by a backward induction.

3 The Policy Competition Stage

3.1 Existence of Equilibrium

Here, we assume that there is a median voter and prove the existence of equilibrium in electoral competition. It is well-known that there may not be a median voter when the policy space is multi-dimensional. We will present a sufficient condition for the existence of a median voter in the next section.

During the voting stage, the median voter compares two candidates by $(p_j;a_j;C_j;p_i;a_i;C_i)$ given the realized valence bias. That is, the voter votes for j over i if and only if

$$v_{(p_m;a_m)}(p_j;a_j;C_j) \quad v_{(p_m;a_m)}(p_i;a_i;C_i) \quad {}_i \quad {}_j$$

where $(p_m;a_m)$ is the median voter's bliss point. Let

$$S_L(p_L;a_L;C_L;p_R;a_R;C_R) \quad \left\{ \in \mathbb{R}^2 | v_{(p_m;a_m)}(p_L;a_L;C_L) \quad v_{(p_m;a_m)}(p_R;a_R;C_R) \quad {}_R \quad {}_L \right\}$$

which is the set of events where the median voter votes for L. Therefore, given $(p_j;a_j;C_j)_{j=L;R}$, the winning probability for j is

$$ {}_L(p_L;a_L;C_L;p_R;a_R;C_R) = \int_{S_L(p_L;a_L;C_L;p_R;a_R;C_R)} g(\ )d\ :$$

Figure 1 depicts the determination of winning probability for a given policy pair.

However, both candidates and IG make their decisions *before* the uncertainty is resolved. Therefore, given the decision in Stage 2, both candidates choose policies to maximize their expected payoff

$$V_j = \ {}_j(p_j;a_j;C_j;p_i;a_i;C_i)w_j^1(p_j;a_j)$$

[17] In a companion paper, Konishi and Pan (2017), we analyze the optimal contract for a single IG extensively.

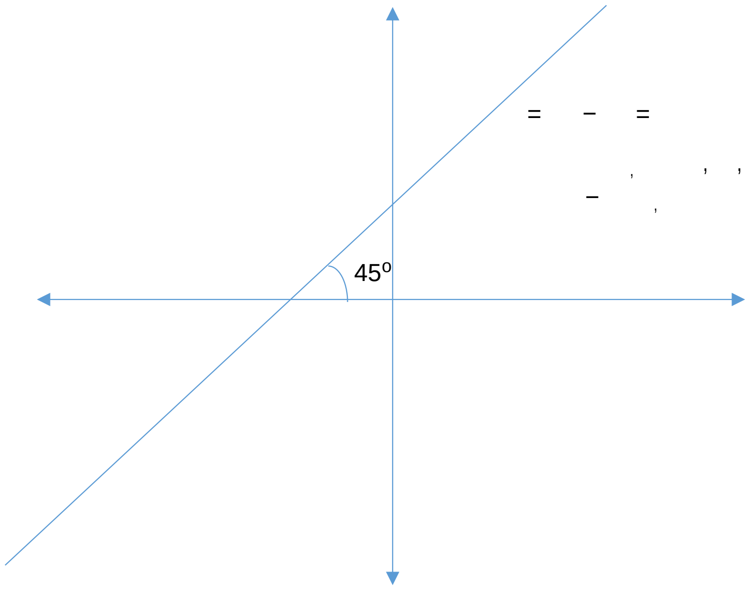

Figure 1: *The winning probability determination for given* $(p_j; a_j; C_j)_{j=L;R}$. *The* 45^o *line stands for the set of events where* L *and* R *are tied.*

The following proposition shows that, under our assumptions on utility functions and the valence density function g, a Nash equilibrium exists. Proposition 1 can be proved as a corollary of Theorem A in Appendix C.[18]

Proposition 1. (Existence) *Suppose that there is a median voter with her bliss point* $(p_m; a_m)$, *and that* $v_{(p_m;a_m)}(p; a; C)$ *and* $w_j^1(p_j; a_j)$ *are quadratic in* $(p; a)$, *and concave in* $(p_j; a_j)$, *respectively. Suppose further that the density function* $g(\ )$ *is log-concave in* $\ 2\ \mathrm{R}^2$. *Then* $_j(p_j; a_j; C_j; p_i; a_i; C_i) w_j^1(p_j; a_j)$ *is log-concave in* $(p_j; a_j)$, *and there exists a Nash equilibrium in policy competition subgame.*

This proposition covers logit model (follows a type-I extreme value distribution). Before concluding this section, we provide another convenient way to represent $_L$ and $_R$. For any $\tilde{\ } 2$ R, define $S_L(\tilde{\ })\ \ f\ 2\ \mathrm{R}^2 j\tilde{\ }\ \ _R\ \ _L g$ and

$$G(\tilde{\ })\ \ \int_{\ 2 S_L(\tilde{\ })} g(\ ) d\ ;$$

Then, candidate L's winning probability is

$$_L(p_L; a_L; C_L; p_R; a_R; C_R) = G\ \ v_{(p_m;a_m)}(p_L; a_L; C_L)\ \ \ v_{(p_m;a_m)}(p_R; a_R; C_R)$$

[18] Theorem A is proved for a Wittman's model (i.e., $\ > 0$) without assuming quadratic utilities (Wittman, 1983).

Similarly,

$$ {}_R(p_R; a_R; C_R; p_L; a_L; C_L) = 1 \quad G \quad v_{(p_m;a_m)}(p_L; a_L; C_L) \quad v_{(p_m;a_m)}(p_R; a_R; C_R) $$

We denote voters' density function by $g(\sim) = \frac{dG}{d\sim}$.

3.2 Symmetric Voter Distribution: Existence of the Median Voter

In the previous subsection, we obtained a general existence result by assuming that there is a median voter in multidimensional policy space. However, it is well-known that we need very strong conditions to assure the existence of the Condorcet winner (Plott 1967) and the existence of the median voter (Davis et al. 1972). Davis et al. (1972) showed that a necessary and su cient condition is that voters' distribution is symmetric in policy space when voters have Euclidean preferences in a voting model without uncertainty. We will provide su cient conditions for the existence of the median voter in our random valence (thus cardinal) model by applying their approach.[19] Voters whose bliss point $(p; a)$ satis es the following condition vote for candidate L.

$$ |p_L \quad p|^2 \quad |a_L \quad a|^2 + C_L + {}_L \quad |p_R \quad p|^2 \quad |a_R \quad a|^2 + C_R + {}_R : \quad (3) $$

Based on the formula above, we can show that voter $(p; a)$ votes for L if

$$ a \quad \frac{1}{2(a_R \quad a_L)} \quad 2(p_R \quad p_L)p + \quad p_R^2 \quad p_L^2 \quad + \quad a_R^2 \quad a_L^2 \quad + (C_L \quad C_R) + {}_R \quad {}_L $$

holds. Figure 2 shows the above line of indi erent voters in the policy space. Note that if the area below the cut-o line in Figure 2 has more voters than the above, candidate L wins. This observation together with a symmetric distribution assumption in Davis et al. (1972), yields the following proposition.

Proposition 2. (Median Voter Result)

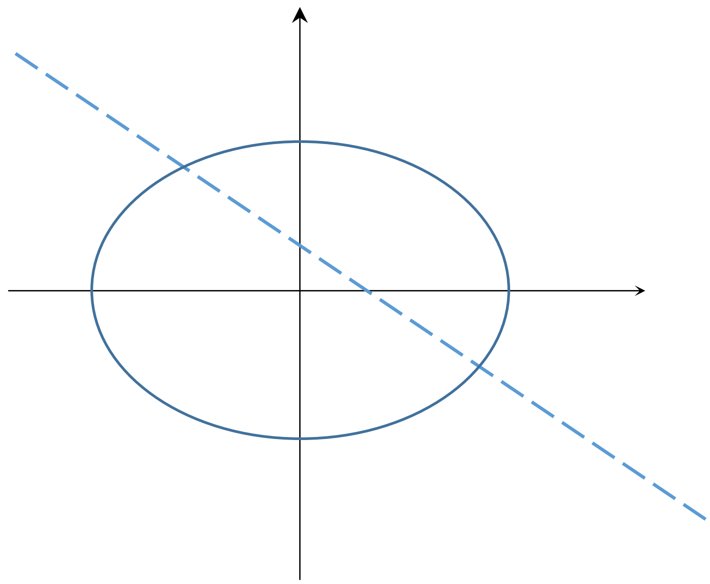

Figure 2: *The cut-o line of indi erent voters.*

Although the \symmetric distribution" assumption (ii) in Proposition 2 is certainly restrictive, it is often employed in the literature of voting problems for multi-dimensional policy spaces.[20] In the rest of the paper (except for the numerical analysis section for the purpose of comparative static analysis), we will normalize the median voter's bliss point at $(0; 0)$ without loss of generality. We will also assume that $a_m = 0 < a_j$ for $j = L; R$, and $p_L < 0(= p_m) < p_R$.

3.3 First-Order Characterization of Equilibrium in Policy Competition Game

Each candidate j's maximization problem is

$$\max_{p_j;a_j} \ _j(p_j; a_j; C_j; p_i; a_i; C) \ fQ + w_p(jp \quad p_j j) + w_a(ja \quad a_j j)g:$$

Naturally assuming $jp_j j > jp_j j$ and $a_j > a_j > 0$ in an equilibrium when candidate j can choose p_j and *date j* ‹573t2date

$j \in \{L,R\}$ with $j \neq i \in \{L,R\}$ are

$$\frac{\partial \;{}_j(p_j,a_j,C_j;p_i,a_i,C_i)}{\partial |p_j|} \quad \{Q + w_p(|\hat{p} - p_j|) + w_a(|\tilde{a} - a_j|)\} \quad {}_j w_p'(|p_j - p_j|) = 0 \tag{4}$$

$$\frac{\partial \;{}_j(p_j,a_j,C_j;p_i,a_i,C_i)}{\partial a_j} \quad \{Q + w_p(|\hat{p} - p_j|) + w_a(|\tilde{a} - a_j|)\} \quad {}_j w_a'(|a_j - a_j|) = 0, \tag{5}$$

where the second equation is omitted when candidate j commits to $a_j = \tilde{a}$. Thus, the Nash equilibrium $(p_j, a_j; p_i, a_i)$ of policy competition is characterized by the above equations (4) and (5) for $i, j \in \{L,R\}$ with $i \neq j$.

3.4 Incentive Compatible Contracts

For simplicity, we assume that IG aims to achieve $\tilde{a}$ no matter which candidate wins by offering C_L and C_R to candidates L and R, respectively. In order to analyze the incentive compatibility of the contracts, let x_j stand for equilibrium x strategy for j *in the subgame that both candidates accept IG's offer.* Also, let x_j stand for the equilibrium policy proposal for j *in the subgame that L rejects the offer.* The L's incentive compatibility (IC) constraint is characterized by

$IC_L \quad {}_L(p_L; \tilde{}f_L$1C.269 0 Td [(L;)-167(R)]TJ/F22 1maT1 Tf 57 6T450 7.9701 Tf 14.478 -0.793 Td |

1. $\frac{d|p_L|}{dC_R} < 0$, $\frac{da_L}{dC_R} < 0$, *and* $\frac{dp_R}{dC_R} > 0$.
2. $\frac{d|p_L|}{da} > 0$, $\frac{da_L}{da} > 0$, *and* $\frac{dp_R}{da} > 0$.
3. *Candidate* L*'s equilibrium payo in this subgame is decreasing in* C_R*.*

Lemma 2. *When both candidates accept* IG*'s o er, comparative static results*

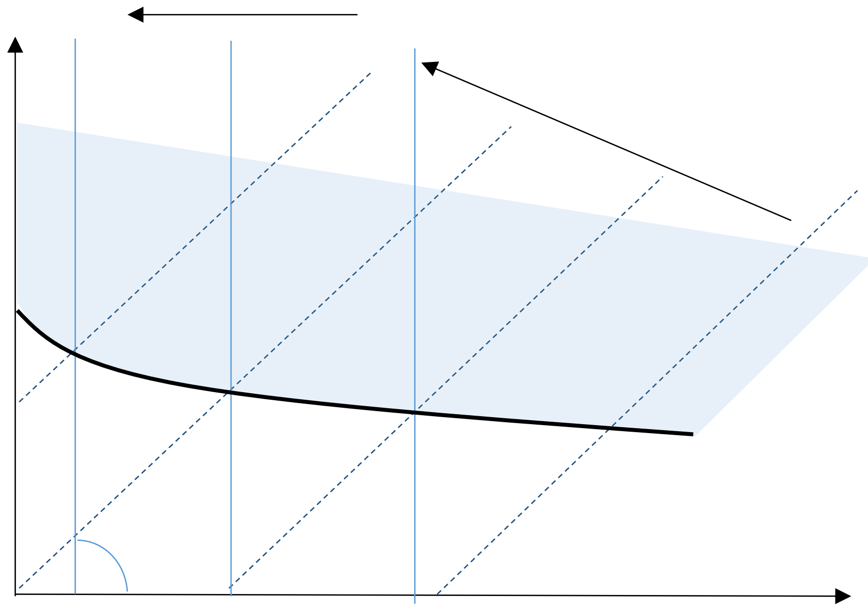

Figure 3:

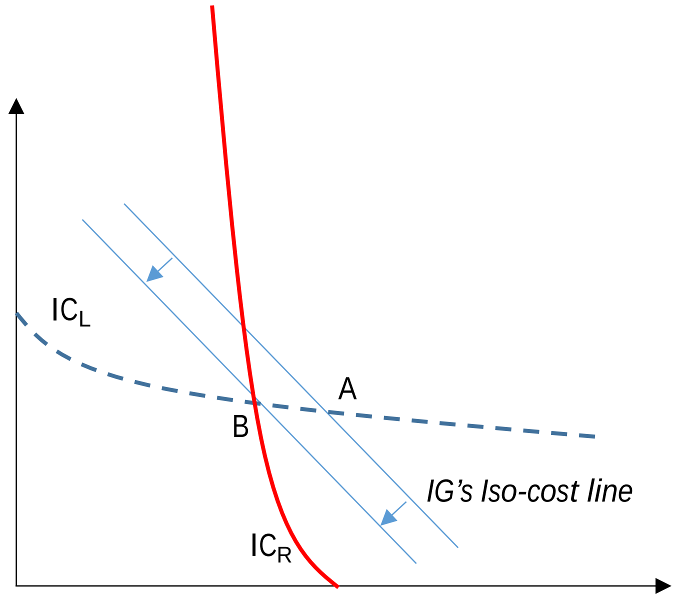

Figure 4: *Binding constraints for both candidates* | *If Regularity in IC Constraints is respected, two constraints can only cross each other once. Moreover, at point A, IC_L is binding, but IC_R is not. IG can reduce its cost by moving along IC_L. At point B, IG's cost is minimized.*

increase in the opponent's campaign contribution do not di er too much between accepting and rejecting IG's o er. We will impose this assumption for the rest of the paper, especially for the asymmetric cases in Section 5.

Proposition 3.

density function at

are asymmetric, to satisfy IC constraints, C_L and C_R need to be adjusted in asymmetric manner in response to an increase in $\bar{a}$. This in turn affects supported equilibrium allocation. The benefit of the symmetry assumption comes from the fact that a symmetric increase in contribution money per se has no direct effect on candidates' policy choice, since candidates care about campaign contributions only when their winning probabilities are affected by them.

5 Numerical Analysis: A Logit Model

Here, we only list the f.o.c's for the equilibrium in which L rejects the o er. Other cases are similar.

$$\frac{\exp(v_{mR})}{\exp(v_{mL}) + \exp(v_{mR})}(p_m \quad p_L) \quad Q \quad (p_L \quad p_L)^2 \quad (a_L \quad a_L)^2 \quad (p_L \quad p_L) = 0$$

$$\frac{\exp(v_{mR})}{\exp(v_{mL}) + \exp(v_{mR})} \ (a_L \quad a_m) \quad Q \quad (p_L \quad p_L)^2 \quad (a_L \quad a$$ (a a

52 Tf 11.928 0 Td [( )]TJ/F17 11.9552 Tf 11.

We demonstrate this by increase $\bar{a}$ from 0.5 to 1. Note that a_L = 0:5 and, as a result, L has minimal incentive to take an extreme ideological position initially. The computational results are listed in Table 1. As we expect from the above argument, L initially moves to the center when $\bar{a}$ is close to a_L but turns back to extreme as $\bar{a}$ becomes larger and larger. Meanwhile, R monotonically moves to his/her own extreme. Therefore, candidates show an asymmetric pattern of polarization in the sense that the more conservative candidate on the agenda becomes more extreme on the ideology dimension as IG becomes more aggressive in promoting the agenda. Moreover, in order to promote $\bar{a}$ more aggressively, IG needs to contribute more to both candidates. It might be surprising that IG contributes more to the candidate who prefers a lower agenda, and this candidate wins more often in the equilibrium. This result is a consequence of the IC constraints: R has a stronger incentive to reject IG. Therefore, IG contributes more to R.[25]

$\bar{a}$	p_L	p_R	C_L	C_R	$_L$
0:5	0:3008	0:3148	0:1706	0:2400	0:4848
0:6	0:2997	0:3187	0:2855	0:3743	0:4808
0:7	0:2994	0:3238	0:4296	0:5389	0:4765
0:8	0:2999	0:3301	0:6026	0:7341	0:4719
0:9	0:3012	0:3377	0:8046	0:9604	0:4669
1:0	0:3033	0:3468	1:0355	1:2186	0:4614

Table 1: Asymmetric equilibrium where $\bar{a}$ 2 [0:5; 1], a_L = 0:5 > 0:3 = a_R.

Before moving on the next example, it is worth pointing out that the candidate who is more reluctant with agenda promotion is also the one proposing a more extreme ideology platform. This is a general trend in our logit example: when a_R decreases, IG contributes more to R

in a raises the incentive for candidates to deviate from accepting the o er *regardless of where the ideology bliss point is.* However, it is not clear how the di erence in contribution money, $C_R \quad C_L$, changes. Our numerical result is shown in Table 2.

a	p_L	p_R	C_L	C_R	$_L$
0:5	0:3201	0:4825	0:1628	0:1657	0:5318
0:6	0:3206	0:4832	0:2781	0:2807	0:5320
0:7	0:3221	0:4851	0:4241	0:4260	0:5324
0:8	0:3247	0:4883	0:6008	0:6014	0:5331
0:9	0:3284	0:4929	0:8084	0:8072	0:5340
1:0	0:3333	0:4989	1:0474	1:0435	0:5354

Table 2: Asymmetric equilibrium where5354

is to see which results would be a ected by assuming Wittman-type candidate utility function. We will start with checking whether or not Propositions 4 and 5 hold in the Wittman setting as long as candidates are symmetric:

$$V_j = \quad_j \, f Q + w_p(jp_j \quad p_j j) + w_a(ja_j \quad a_j j) g + (1 \quad_j) \quad f w_p(jp_i \quad p_j j) + w_a(ja_i \quad a_j j) g;$$

where < 1. In a symmetric equilibrium $p_j = \quad p_i$ when $a_j = a_i = a$, we have

$$V_j = \quad_j \quad Q + w_p(\, p_j \quad p_j \,) + w_a(ja \quad a_j j) \; + (1 \quad_j) \quad f w_p(jp_i \quad p_j j) + w_a(ja \quad a_j j) g: \quad jp_j$$

\pdfa3 F50 7.9701 Tf 8.367 -1.793 Td [(p)]TJ/F17 11.

since $a >$

The rst order condition with respect to p_j at a symmetric equilibrium is

$$4'(0) \; p_j \quad Q + w_p(\, p_j \quad p_j \,) + (1 \quad) \, w_a(ja \quad a_j j)$$
$$w_p(\quad p_j \quad p_j \,) \quad w_p^0(\, p_j \quad p_j \,) = 0:$$

Thus, as long as < 1, the contents of the bracket goes down by an increase of a since $a > a_j$, and p_j approaches to p_j, causing polarlization (Proposition 5).[27] In contrast, Proposition 4 may not hold when is large enough. In the proof of Proposition 4, we use the property that the LHS of the above decreases monotonically as p_j increases. However, an additional term $w_p(jp_i \quad p_j j)$ may dominate $w_p(\, p_j \quad p_j \,)$ when is large, and the uniquess of symmetric equilibrium may not be assured in this setup.

We will also conduct numerical analysis for the Wittman case to show the robustness of our results. We test the robustness by considering > 0 in our logit model:

Candidate j's expected utility is

$$W_j(p_j; a_j; p_i; a_i) = \qquad \exp(v_{mj})$$

= 0			= 0.5			= 0.9		
a	$\lvert p_L \rvert = p_R$	C	a	$\lvert p_L \rvert = p_R$	C	a	$\lvert p_L \rvert = p_R$	C
0.5	0.3068	0.1660	0.5	0.2752	0.1642	0.5	0.2546	0.1632
0.75	0.3160	0.5052	0.75	0.2780	0.4879	0.75	0.2550	0.4774
1	0.3300	1.0311	1	0.2831	0.9804	1	0.2559	0.9510

Table 3: Symmetric equilibrium policies and offers when = 0; 0.5 and 0.9.

For the asymmetric equilibrium in which $a_L > a_R$, the same intuition applies. Since the incentive of polarization is weaker when is higher, we expect that, if a is in a relatively lower range, the effect of increasing C_R C_L should dominates more often. In the following table, we use the same parameter as what in Table 1, but we set = 0.5.

a	p_L	p_R	C_L	C_R	$_L$
0.5	0.2703	0.2795	0.1162	0.1642	0.4893
0.6	0.2691	0.2818	0.2102	0.2738	0.4859
0.8	0.2677	0.2877	0.4804	0.5763	0.4788
0.9	0.2675	0.2913	0.6559	0.7692	0.4750
0.95	0.2674	0.2933	0.7537	0.8737	0.4731
1.0	0.2675	0.2955	0.8580	0.9895	0.4711

Table 4: Asymmetric equilibrium policies and offers when = 0.5 and $a_L = 0.5 > 0.3 = a_R$.

Note that, in contrast to the case in Table 1 (where = 0), L goes more moderate up to $a \simeq 0.95$. This can be compared with Table 1 where L turns back to the extreme around $a = 0.7 < 0.95$. It is again the asymmetric polarization pattern we expect to see.

Expected Utility Maximizing IG

We simplified our model by assuming that IG has a target agenda level a, and what it does is to minimize the cost to achieve that goal. Obviously this is a restrictive assumption. This setup can be justified by assuming an extreme risk-averse IG promoting its optimal agenda a. Suppose that IG has a strictly concave von Neumann-Morgenstern utility function $u(a)$. Then, IG's problem can be set up as follows:

$$\max_{(a_L;a_R)} \quad {}_L u(a_L) + \quad {}_R u(a_R) \quad C_L \quad C_R$$

subject to IC_L 0 and IC_R 0.

When two candidates are symmetric, then there will be $a_L = a_R = a$ with $C_L = C_R$ that maximizes IG's expected utility. In that allocation, cost minimization must be achieved for IG, so there is no difference in the first-order characterization of the optimum. Since equilibrium a increases by IG's getting stronger preference for higher a, our Proposition 5 says that, in equilibrium, as IG gets stronger preference for a, polarization happens and contributions surge.

When the candidates are not symmetric, $a_L = a_R = a$

It is easily imaginable that as it becomes more expensive to get two candi-

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Appendix A: Incentives for Exporting Firms to Make Campaign Contributions

The main bene ciaries of free trade are clearly exporting rms. If trade barriers by foreign countries are reduced, they can increase exports and pro ts tremendously. However, these countries have no reason to reduce their tari s unilaterally for the US. They also want to protect their domestic rms. This was precisely the reason that the Reciprocal Trade Agreement Act (RTAA) was passed in 1934. In the early 1930s, high tari s caused by the Smoot-Hawley Act contributed to the downward spiral of trade as other countries retaliated against the United States. Passing RTAA, Congress e ectively gave up control over the US tari s, authorizing President Franklin Roosevelt to enter into tari agreements with foreign countries to reduce import duties in order to speed the recovery from the Depression.[29] Irwin (2015) argues: \The RTAA explicitly linked foreign tari reductions that were bene cial to exporters to lower tari protection for producers competing against imports. This enabled exporters to organize and oppose high domestic tari s because they want to secure lower foreign tari s on their products." (Irwin, 2015, pp. 242) After World War II, the General Agreement on Tari s and Trade (GATT) broadened the tari negotiation talks to a multilateral system under the \reciprocity" and \nondiscrimination" principles, through the `most-favored-nation' (MFN) clause (Bagwell and Staiger, 1999).[30] RTAA and GATT helped to bolster the lobbying position of exporters in the political process, and expanding trade through tari reductions increased the size of strong industries and decreased the size of import competing industries (Irwin, 2015). As long as negotiation tables with other countries are set up and a good negotiation team is appointed, exporting rms can lobby for lowering the tari rates. Thus, exporting rms have incentives to make campaign contributions to (possibly both) presidential candidates as to keep free trade/globalization issue nonsalient.[31]

Reciprocity is one of the key principles of international negotiations in tari reductions in GATT and preferential trade agreements (Bagwell and Staiger 1999). For exporting rms to enjoy low foreign tari rates, the home country also needs to reduce its tari rates. Otherwise, the negotiation will not be quasi-linear product di erentiation model by Melitz and Ottaviano (2008), reciprocity in two-country trade negotiation is analyzed (Bagwell and Staiger, 1999). Kim (2017) shows that productive exporting rms are more likely to lobby for reduced tari s than less productive rms when products are more di erentiated, and he provides empirical evidences for his predictions. He obtains this result by employing the protection-for-sale model in Grossman and Helpman (1994) as a proxy of the tari negotiation process between two countries, assuming that the countries are symmetric.

Kim's paper shows that as long as countries are at the negotiation table for trade deals, productive exporting rms can lobby hard for lower tari s for their products, gaining access to large foreign markets.[32] However, the presence of international negotiation tables is not always assured, as with the tari wars in early 1930s. Without a negotiation table, exporting rms have no way to lobby for lower tari rates levied by foreign countries. GATT provided this service with the principles of reciprocity and most favored nations clause (MFN), and preferential trade agreements such as NAFTA, TPP, and TTIP provide additional negotiation tables.[33] Thus, it is indeed in exporting rms' interests to have a president who is willing to commit to promoting free trade.

Appendix B: The 2016 Presidential Race

Recently, we can observe an increasing trend of negative sentiments toward globalism in the US and other Western countries. Autor, Dorn, and Hanson (2013) report that the rise of competition with China and other developing countries explains 25% of the decline in US manufacturing employment between 1990 and 2007.[34] In the 2016 US presidential campaign, anti-globalism/protectionism became one of the most salient issues, and industries' contributions to the two party nominees showed quite di erent patterns relative to prior presidential election years. In prior years, for almost all sectors/industries, the top two recipients of campaign contributions are most likely to be the Republican and Democratic party nominees, but in the 2016 presidential election race, Donald Trump received signi cantly lower contributions from industries that have interests in trade agreements.

The Center of Responsive Politics provides detailed information on US politics (https://www.opensecrets.org/). We can get information on sector/industry-level contributions to each candidate who ran in presidential races (detailed decompositions are available from at least 2008 on). Each sector/industry provides contributions to a number of candidates including both parties' presidential nominees and other candidates who drop out as party primaries proceed. Sector/industries often have a party bias.

2008	1	2	3	Obama	

2016

Appendix C: Electoral Competition

In this part, we shall provide a general existence result of electoral equilibrium in a two-party setting by assuming that there is a single voter (or a median voter) in K-dimensional policy space. By a slight abuse of notation, we denote a policy as $p = (p^1; p^2; \ldots; p^K) \in \mathbb{R}^K$ instead of $(p; a)$ in this subsection. Here, we will set up a version of the Wittman model with valence (Wittman 1983). Following Wittman, we assume that candidate j's payoff function is

$$V_j(p_j; p_i) = \pi_j(p_j; p_i) w_j^1(p_j) + (1 - \pi_j(p_j; p_i)) w_j^0(p_i);$$

where $w_j^1(p_j)$ and $w_j^0(p_i)$ are candidate j's payoffs when she wins or loses an election, respectively. By setting $w_j^0(p_i) = 0$ for all p_i, the theorem below covers Proposition 1 as a special case. We also drop C_js from the voter's utility function since C_js are fixed here. During the voting stage, voters compare two candidates by p_j and p_i given the realized valence bias. That is, the median voter votes for $j \in \{L; R\}$ over $i \in \{L; R\}$ with $i \neq j$ if and only if

$$v(|p_j - p_m|) - v(|p_i - p_m|) \geq \varepsilon_i - \varepsilon_j;$$

where ε_j denotes a random valence term for candidate j. Let

$$S_j(p_j; p_i) \equiv \left\{ \varepsilon \in \mathbb{R}^2 \,|\, v(|p_j - p_m|) - v(|p_i - p_m|) \geq \varepsilon_i - \varepsilon_j \right\}$$

which is the set of events where the pivotal voter votes for j. Note that $S_j(p_j; p_i)$ is a convex set in $\mathbb{R}^2$. Therefore, the winning probability for j is

$$\pi_j(p_j; p_i) = \int_{S_j(p_j; p_i)} g(\varepsilon) d\varepsilon :$$

The following mathematical result is useful in proving the existence of equilibrium.

The Prekopa Theorem (Prekopa 1973). *Let μ be a probability density function on $\mathbb{R}^K$ with convex support C. Take any measurable sets A_0 and A_1 in $\mathbb{R}^K$ with $A_0 \cap C \neq \emptyset$ and $A_1 \cap C \neq \emptyset$. For any $0 \leq \lambda \leq 1$, define $A_\lambda = (1 - \lambda) A_0 + \lambda A_1$, the Minkowski average of the two sets.*[35] *If $\mu(\cdot)$ is log concave, then*

$$\log \int_{A_\lambda} \mu(x) dx \geq (1 - \lambda) \log \int_{A_0} \mu(x) dx + \lambda \log \int_{A_1} \mu(x) dx :$$

We prove the following theorem by utilizing the Prekopa theorem:

Theorem A. (Existence) *Let $P_j \subset \mathbb{R}^K$ be a compact and convex policy space. Suppose that there is a median voter, and that $v(|p_j - p_m|)$ and $w_j^1(p_j)$ are continuous and concave in p_j, respectively, $w_j^0(p_i)$*

[35] The Minkowski average A_λ is defined as all points of the form $x_\lambda = (1 - \lambda) x_0 + \lambda x_1$, with $x_0 \in A_0$, $x_1 \in A_1$, and $0 \leq \lambda \leq 1$.

is continuous in p_i, and the density function $g(\theta)$ is log-concave in $\theta \in \mathbb{R}^2$. Then, there exists a Nash equilibrium in the policy competition subgame.

Proof. Since C_L and C_R are fixed in this proposition, we will drop them from u_m's arguments. Since v is concave, note that for all p_j, p_j', and all $\lambda \in [0;1]$,

$$v(|\lambda p_j + (1-\lambda)p_j' - p_m|) \geq \lambda v(|p_j - p_m|) + (1-\lambda)v(|p_j' - p_m|)$$

By Prekopa's theorem (Prekopa 1973), we have

$$\int_{\lambda S(p_j;p_i)+(1-\lambda)S(p_j';p_i)} g(\theta)d\theta \geq \lambda \int_{S(p_j;p_i)} g(\theta)d\theta + (1-\lambda)\int_{S(p_j';p_i)} g(\theta)d\theta$$

Now, by definition of S_j and concavity of v, we have

$$S(\lambda p_j + (1-\lambda)p_j';p_i) \supseteq \lambda S(p_j;p_i) + (1-\lambda)S(p_j';p_i):$$

This implies

$$\int_{S(\lambda p_j+(1-\lambda)p_j';p_i)} g(\theta)d\theta \geq \int_{\lambda S(p_j;p_i)+(1-\lambda)S(p_j';p_i)} g(\theta)d\theta\ ;$$

and

$$\int_{S(\lambda p_j+(1-\lambda)p_j';p_i)} g(\theta)d\theta \geq \lambda\int_{S(p_j;p_i)} g(\theta)d\theta + (1-\lambda)\int_{S(p_j';p_i)} g(\theta)d\theta\ :$$

Therefore, we conclude that $\psi_j(p_j;p_i) = \int_{S(p_j;p_i)} g(\theta)d\theta$ is log-concave in p_j if g is log-concave in θ.

Let candidate j's best response $\phi_j : P_i \rightrightarrows P_j$ be such that

$$\phi_j(p_i) \equiv \arg\max_{p_j \in P_j} V_j(p_j;p_i)$$

This correspondence is nonempty-valued and upper hemicontinuous (continuity of V_j).

Using a trick by Roemer (1997), we can rewrite candidate j's payoff function in a convenient way:

$$V_j(p_j;p_i) = \psi_j(p_j;p_i)\left(w_j^1(p_j) - w_j^0(p_i)\right) + w_j^0(p_i).$$

Thus, we have

$$\log\left(V_j(p_j;p_i) - w_j^0(\right.$$

Remark 1. Proposition 1 is a special case of this theorem ($w_j^0 = 0$, and v_j^m is quadratic). If the policy space is one-dimensional, then there exists a median voter, and thus Theorem A guarantees the existence of electoral competition. Note that this theorem shows existence of equilibrium when uncertainty is generated only by valence terms. Roemer (1997) and Duggan and Martinelli (2017) use a model with uncertain median voter's position, which behaves di erently, making the best response correspondence potentially discontinuous or nonconvex-valued. Note also that Duggan and Martinelli (2017) assumes log concavity of G. Here we assume a stronger condition: log concavity of g.

Proof of Proposition 2. The two candidates' policies are $(p_L; a_L; C_L)$ and $(p_R; a_R; C_R)$. Suppose that we have

$$a_m \quad \frac{1}{2\,(a_R \quad a_L)} \quad 2\,(p_R \quad p_L)\,p_m + \; p_R^2 \quad p_L^2 \; + \; a_R^2 \quad a_L^2 \; + (C_L \quad C_R) + \;{}_R \quad {}_L$$

Then, in $(p; a)$-space, $(p_m; a_m)$ is below the voting cut-o line, the voter $(p_m; a_m)$ votes for candidate L, who receives more votes than candidate R

we have

$$\frac{\frac{\partial \pi_L}{\partial |p_L|}(p_L, a_L, 0, p_R, \bar{a}, C_R)}{\pi_L(p_L, a_L, 0, p_R, \bar{a}, C_R)} = \frac{\bar{g}(v_L - v_R)}{\bar{G}(v_L - v_R)} v'_p(|p_L|)$$

$$\frac{\frac{\partial \pi_L}{\partial a_L}(p_L, a_L, 0, p_R, \bar{a}, C_R)}{\pi_L(p_L, a_L, 0, p_R, \bar{a}, C_R)} = \frac{\bar{g}(v_L - v_R)}{\bar{G}(v_L - v_R)} v'_a(|a_L|)$$

$$\frac{\frac{\partial \pi_R}{\partial p_R}(p_R, \bar{a}, C_R, p_L, a_L, 0)}{\pi_R(p_R, \bar{a}, C_R, p_L, a_L, 0)} = \bar{g}(v_R - v_L)$$

where $v'_{pj} = v'_p(|p_j|) = 2|p_j|$, $v''_{pj} = v''_p(|p_j|) = 2$, $v'_{aL} = v'_a(|a_L|) = 2|a_L|$, $v''_{aL} = v''_a(|a_L|) = 2$, $v'_{\hat{a}R} = v'_a(|\hat{a}|) = 2|\hat{a}|$, $w'_{pj} = w'_p(|p_j \quad p_j|)$, $w'_{aL} = w'_a(|a_L \quad a_L|)$, $w'_{\hat{a}R} = w'_a(|\hat{a} \quad a_R|)$, $'_L = '(\quad)$, $'_R = '(\quad)$, and we drop all double-asterisk superscripts for conciseness. Denoting the LHS matrix by D, we can show that the determinant of D has a positive sign.

For the derivations of the next two lemmas, please refer Technical Appendix.

Lemma A1. $|D| > 0$.

With Lemma A1, we can conduct comparative static exercises.

Lemma 1. *In the subgame where candidate L rejects the o er, comparative static results on the Nash equilibrium of policy competition are:*

1. $\frac{d|p_L|}{dC_R} < 0$, $\frac{da_L}{dC_R} < 0$, $\frac{dp_R}{dC_R} > 0$, *and* $\frac{d}{dC_R} < 0$.
2. $\frac{d|p_L|}{d\hat{a}} > 0$, $\frac{da_L}{d\hat{a}} > 0$, *and* $\frac{d}{d\hat{a}} > 0$, *and* $\frac{dp_R}{d\hat{a}} > 0$.
3. *Candidate L's equilibrium payo in this subgame is decreasing in C_R.*

The case where candidate R rejects the o er is symmetrically analyzed.

Equilibrium when both candidates accept the o er

Letting $= v_L \quad v_R = v_p(|p_L|) + v_a(|\hat{a}|) + C_L \quad v_p(|p_R|) \quad v_a(|\hat{a}|) \quad C_R$, the system of equation that characterizes the equilibrium in this case is written as

$$'(\quad)v'_{pL}\, \{Q + w_p(|p_L \quad p_L|) + w_a(|a_L \quad a_L|)\} \quad) \quad ^{d1}{}_L \qquad (1.$$

For the derivations of the next two lemmas, please refer Technical Appendix.

Lemma A2. $\hat{D} < 0$

We conduct comparative statics in this case, too.

Lemma 2. *When both candidates accept IG's o er, comparative static results on policy competition equilibrium are:* $\frac{d|p_L|}{da} > 0$, $\frac{dp_R}{da} > 0$, $\frac{d|p_L|}{dC_L} > 0$, $\frac{dp_R}{dC_L} < 0$, $\frac{d}{dC_L} > 0$, $\frac{d|p_L|}{dC_R} < 0$, $\frac{dp_R}{dC_R} > 0$, $\frac{d}{dC_R} < 0$. *Moreover, L's equilibrium payo in this subgame is decreasing in* C_R *and increasing in* C_L*.*

Proof of Proposition 3. First, note that $\frac{dC_L}{dC_R}\Big|_{IC_L=0} < 1$ holds from (8). This is because $\frac{d(\ {}_L w_L{}^{L}{}_{R})}{dC_R}$

Since $Q + w_p(|p_L \quad p_L|) + w_a(|a \quad a_L|)$ goes down, the LHS of the above IC constraint decreases as a increases.

In contrast, without adjustment in C, the contents of the RHS is increased by an increase of a:

$$\frac{dRHS}{da} = g(\quad)\frac{d}{da}w_L + G(\quad) \quad w'_{pL}\frac{d|p_L|}{da}$$

Table A4: Increasing trend of protectionism — a_m decreases from 0.3 to 0.

Again, we observe an asymmetric polarization. The Republican's ideological position polarizes as a_m goes down, while the Democrat's position does not change much and even moves toward center slightly. Thus, if Republican candidates are more reluctant to promote free trade than Democrat's, then the asymmetric polarization can be explained by the increasing trend of protectionism.[36]

Ex Ante Valence Advantage

In the benchmark case, we assume that the voter is *unbiased* toward the two candidates in the sense that, as long as the policy proposals and campaign contributions are symmetric, the winning probability is also the same. However, it is often the case that one candidate may have a "non-policy" advantage, such as incumbency or strong personal charisma. To incorporate this effect, we assume the voters evaluate L and R by

$$v(|p_L - p_m|, |a_L - a_m|, C_L) + \varepsilon_L + \delta,$$
$$v(|p_R - p_m|, |a_R - a_m|, C_R) + \varepsilon_R,$$

where δ stands for a nonrandom advantage that L has at the beginning of the election (a disadvantage if δ is negative). It is relatively straightforward to show that, in the equilibrium where both candidates accept some advantages even at small positive , since her/his preference is more in line with the voter. In this situation, candidate R's policy position is most polarized while L's position moves slightly towards the center. Notice that p_L and p_R move in the same direction as the symmetric case.[37]

	p_L	p_R	C_L	C_R	$_L$
0:2	0:2839	0:3514	0:6035	0:7431	0:4264
0:1	0:2917	0:3404	0:6031	0:7384	0:4491
0	0:2999	0:3301	0:6026	0:7341	0:4719
0:1	0:3087	0:3203	0:6022	0:7303	0:4948
0:2	0:3181	0:3112	0:6018	0:7268	0:5177

Table A6: Shifting ex ante advantage from one candidate to the other | $a_L > a_R$ case.

This result is in stark contrast with the one in Groseclose (2001), which shows that the advantageous candidate moves toward the center while the disadvantageous candidate moves away from the center when one candidate has a small advantage. Unlike our uncertain valence model, the source of uncertainty is from the median voter's position in Groseclose (2001). In his model, the median voter's position can be very sensitive to proposed policies when the utility function has high curvature and the ex ante advantage is small. Therefore, it is possible that the advantageous candidate proposes a more central policy under such a situation. This suggests that di erent ways to incorporate uncertainty have distinct comparative statics.[38]

[37] Chamon and Kaplan (2013) also consider the ex ante valence advantage in their framework. Similar to our result, they conclude that more contributions go to the advantageous candidate.

[38] Our result can also be seen as a theoretical base for a so-called marginality hypothesis, that is, electoral competition increases responsiveness on policy. (Fiorina, 1973). The empirical evidence of this hypothesis is mixed depending on how the valence advantage is de ned. Recent supporting evidence includes Ansolabehere, Snyder, and Steward (2001) and Gri n (2006).

Technical Appendix (Not for Publication)

Here, we collect technical derivations of Appendix A.

Lemma A1. $jDj > 0$.

Proof of Lemma 1. Direct calculations.

$$jDj = v^0_{pL} \begin{array}{ccc} '_L v^0_{pL} w^0_{aL} & 0 & '^0_L v^0_{pL} w_L \\ '_L (v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L) + w^{00}_{aL} & 0 & '^0_L v^0_{aL} w_L \\ 0 & '_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R + w^{00}_{pR} & '^0_R v^0_{pR} w_R \end{array}$$

$$+ v^0_{aL} \begin{array}{ccc} '_L \; v^0_{pL} w^0_{pL} + v^{00}_{pL} w_L + w^{00}_{pL} & 0 & '^0_L v^0_{pL} w_L \\ '_L v^0_{aL} w^0_{pL} & 0 & '^0_L v^0_{aL} w_L \\ 0 & '_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R + w^{00}_{pR} & '^0_R v^0_{pR} w_R \end{array}$$

$$+ v^0_{pR} \begin{array}{ccc} '_L \; v^0_{pL} w^0_{pL} + v^{00}_{pL} w_L + w^{00}_{pL} & '_L v^0_{pL} w^0_{aL} & '^0_L v^0_{pL} w_L \\ '_L v^0_{aL} w^0_{pL} & '_L (v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L) + w^{00}_{aL} & '^0_L v^0_{aL} w_L \\ 0 & 0 & '^0_R v^0_{pR} w_R \end{array}$$

$$\begin{array}{ccc} '_L \; v^0_{pL} w^0_{pL} + v^{00}_{pL} w_L + w^{00}_{pL} & '_L v^0_{pL} w^0_{aL} & 0 \\ '_L v^0_{aL} w^0_{pL} & '_L (v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L) + w^{00}_{aL} & 0 \\ 0 & 0 & '_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R + w^{00}_{pR} \end{array}$$

$$= \; '_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R + w^{00}_{pR}$$

$$+ \; n \; v^0_{pL} \begin{array}{cc} '_L v^0_{aL} w^0_{pL} & '^0_L v^0_{pL} w_L \\ '_L (v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L) + w^{00}_{aL} & '^0_L v^0_{aL} w_L \end{array} \; v^0_{aL} \begin{array}{cc} '_L \; v^0_{pL} w^0_{pL} + v^{00}_{pL} w_L + w^{00}_{pL} & '^0_L v^0_{pL} w_L \\ '_L v^0_{aL} w^0_{pL} & '^0_L v^0_{aL} w_L \end{array} \; o$$

$$'_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \quad w^{00}_{pR} \quad '^0_R \; v^0_{pR} \; {}^2 w_R$$

$$\begin{array}{cc} '_L \; v^0_{pL} w^0_{pL} + v^{00}_{pL} w_L + w^{00}_{pL} & '_L v^0_{pL} w^0_{aL} \\ '_L v^0_{aL} w^0_{pL} & '_L (v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L) + w^{00}_{aL} \end{array}$$

$$= \; '_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R + w^{00}_{pR}$$

$$n \; '_L \, '^0_L \; v^0_{pL} \; {}^2 v^{00}_{aL} w^2_L \quad '^0_L \; v^0_{pL} \; {}^2 w_L w^{00}_{aL} \quad '_L \, '^0_L (v^0_{aL})^2 v^{00}_{pL} w^2_L \quad '^0_L (v^0_{aL})^2 w_L w^{00}_{pL} \; o$$

$$+ \; n \; '_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \quad w^{00}_{pR} \quad '^0_R \; v^0_{pR} \; {}^2 w_R \; o$$

$$'^2_L w_L \;\; v^{00}_{pL} v^{00}_{aL} w_L \quad v^0_{pL} w^0_{pL} v^{00}_{aL} \quad v^0_{aL} w^0_{aL} v^{00}_{pL} \; + w^{00}_{pL} w^{00}_{aL}$$

$$+ \; '_L \; v^0_{pL} w^0_{pL} + v^{00}_{pL} w_L \;\; w^{00}_{aL} + \; '_L (\; v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L) w^{00}_{pL}$$

$$> 0$$

We have completed the proof.

Now, we are ready to conduct comparative static exercises.

Lemma 1. When candidate L rejects the o er, the comparative static results on policy competition are:

1. $\frac{djp_L j}{dC} < 0$, $\frac{da_L}{dC} < 0$, $\frac{dp_R}{dC} > 0$, and $\frac{d}{dC} < 0$.

2. $\frac{djp_L j}{d\bar{a}} > 0$, $\frac{da_L}{d\bar{a}} > 0$, and $\frac{d}{d\bar{a}} > 0$, and $\frac{dp_R}{d\bar{a}}$ R 0.

3. *Candidate L*

$$\frac{d\,j p_L j}{da} = \frac{1}{jDj}\begin{array}{cccc} 0 & {}'_L v^0_{pL} w^0_{aL} & 0 & {}'^0_L v^0_{pL} w_L \\ 0 & {}'_L \left(v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L \right) + w^{00}_{aL} & 0 & {}'^0_L v^0_{aL} w_L \\ {}'_R w^0_{aR} v^0_{pR} & 0 & {}'_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \; + w^{00}_{pR} & {}'^0_R v^0_{pR} w_R \\ v^0_{aR} & v^0_{aL} & v^0_{pR} & 1 \end{array}$$

$$= \frac{{}'_R w^0_{aR} v^0_{pR}}{jDj}\begin{array}{ccc} {}'_L v^0_{pL} w^0_{aL} & 0 & {}'^0_L v^0_{pL} w_L \\ {}'_L \left(v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L \right) + w^{00}_{aL} & 0 & {}'^0_L v^0_{aL} w_L \\ v^0 a_L & v^0_{pR} & 1 \end{array}$$

$$\frac{v^0_{aR}}{jDj}\begin{array}{ccc} {}'_L v^0_{pL} w^0_{aL} & 0 & {}'^0_L v^0_{pL} w_L \\ {}'_L \left(v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L \right) + w^{00}_{aL} & 0 & {}'^0_L v^0_{aL} w_L \\ 0 & {}'_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \; + w^{00}_{pR} & {}'^0_R v^0_{pR} w_R \end{array}$$

$$= \frac{{}'_R w^0_{aR} \; v^0_{pR} \;^2 + v^0_{aR} \; {}'_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \; + w^{00}_{pR}}{jDj}$$

$${}'^0_L v^0_{pL} w_L \; f \, {}'_L v^{00}_{aL} w_L + w^{00}_{aL} g \; > 0$$

$$\frac{da_L}{da} = \frac{1}{jDj}\begin{array}{cccc} {}'_L \; v^0_{pL} w^0_{pL} + v^{00}_{pL} w_L \; + w^{00}_{pL} & 0 & 0 & {}'^0_L v^0_{pL} w_L \\ {}'_L v^0_{aL} w^0_{pL} & 0 & 0 & {}'^0_L v^0_{aL} w_L \\ 0 & {}'_R w^0_{aR} v^0_{pR} & {}'_R \; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \; + w^{00}_{pR} & {}'^0_R v^0_{pR} w_R \\ v^0_{pL} & v^0_{aR} & v^0_{pR} & 1 \end{array}$$

$$= \frac{{}'_R w^0_{aR} v^0_{pR}}{jDj}\begin{array}{ccc} {}'_L \; v^0_{pL} w^0_{pL} + v^{00}_{pL} w_L \; + w^{00}_{pL} & 0 & {}'^0_L v^0_{pL} w_L \\ {}'_L v^0_{aL} w^0_{pL} & 0 & {}'^0_L v^0_{aL} w_L \\ v^0_{pL} & v^0_{pR} & 1 \end{array}$$

$$\frac{dp_R}{da} = \begin{matrix} '_L \quad v^{0}_{pL} w^{0}_{pL} + v^{00}_{pL} w_L \quad + w^{00}_{pL} & '_L v^{0}_{pL} w^{0}_{aL} & 0 & '^{0}_L v^{0}_{pL} w_L \\ '_L v^{0}_{aL} w^{0}_{pL} & '_L (\quad v_{aL} w^{0}_{aL} + v^{00}_{aL} w_L) + w^{00}_{aL} & 0 & '^{0}_L v \end{matrix}$$

$$
\frac{djp_L j}{da_m} = \frac{1}{jDj}
\begin{matrix}
0 & {}'_L v^0_{pL} w^0_{aL} & 0 & {}'^0_L v^0_{pL} w_L \\
{}'_L v^{00}_{aL} w_L & {}'_L (\; v^0_{aL} w^0_{aL} + v^{00}_{aL} w_L) + w^{00}_{aL} & 0 & {}'^0_L v^0_{aL} w_L \\
0 & 0 & {}'_R \;\; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \;\; + w^{00}_{pR} & {}'^0_R v^0_{pR} w_R \\
v^0_{aL} \;\; v^0_{aR} & v^0_{aL} & v^0_{pR} & 1
\end{matrix}
$$

$$
= \frac{1}{jDj}(\;\; {}'_R \;\; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \;\; w^{00}_{pR} \;\; {}'^0_R (v^0_{pR})^2 w_R)({}'^2_L v^0_{pL} w^0_{aL} v^{00}_{aL} w_L)
$$

$$
+ \frac{1}{jDj} v^0_{aL} ({}'_R \;\; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \;\; + w^{00}_{pR})({}'_L \; {}'^0_L v^{00}_{aL} v^0_{pL} w^2_L)
$$

$$
+ \frac{1}{jDj} (v^0_{aL} \;\; v^0_{aR})({}'_R \;\; v^0_{pR} w^0_{pR} + v^{00}_{pR} w_R \;\; + w^{00}_{pR})(\;\; {}'^0_L v^0_{pL} w_L)({}'_L v^{00}_{aL} w_L \; + w^{00}_{aL})
$$

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d

$$\frac{dp_R}{da} = \frac{1}{\hat{D}} \begin{vmatrix} {}'_L \quad v'_{pL} w'_{pL} + v''_{pL} w_L \quad + w''_{pL} & {}'_L w'_{aL} v'_{pL} & {}'^{0}_L v'_{pL} w_L \\ 0 & {}'_R w'_{aR} v'_{pR} & {}'^{0}_R v'_{pR} w_R \\ v'_{pL} & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{\hat{D}} f\, {}'_R w'_{aR} v'_{pR} [\, {}'_L \quad v'_{pL} w'_{pL} + v''_{pL} w_L \quad + w''_{pL}]$$

$$+ v'_{pL} [\, {}'_L w'_{aL} v'_{pL} \; {}'^{0}_R v'_{pR} w_R + \; {}'_R w'_{aR} v'_{pR} \; {}'^{0}_L v'_{pL} w_L] g > 0$$

$$\frac{d\, j p_L j}{dC_L} = \frac{1}{\hat{D}} \begin{vmatrix} 0 & 0 & {}'^{0}_L v'_{pL} w_L \\ 0 & {}'_R \quad v'_{pR} w'_{pR} + v''_{pR} w_R \quad + w''_{pR} & {}'^{0}_R v'_{pR} w_R \\ 1 & v'_{pR} & 1 \end{vmatrix}$$

$$= \frac{{}'^{0}_L v'_{pL} w_L \quad {}'_R \quad v'_{pR} w'_{pR} + v''_{pR} w_R \quad + w''_{pR}}{\hat{D}} > 0$$

$$\frac{dp_R}{dC_L} = \frac{1}{\hat{D}} \begin{vmatrix} {}'_L \quad v'_{pL} w'_{pL} + v''_{pL} w_L \quad + w''_{pL} & 0 & {}'^{0}_L v'_{pL} w_L \\ 0 & 0 & {}'^{0}_R v'_{pR} w_R \\ v'_{pL} & 1 & 1 \end{vmatrix}$$

$$= \frac{{}'^{0}_R v'_{pR} w_R \quad {}'_L \quad v'_{pL} w'_{pL} + v''_{pL} w_L \quad + w''_{pL}}{\hat{D}} < 0$$

$$\frac{d}{dC_L} = \frac{1}{\hat{D}} \begin{vmatrix} {}'_L \quad v'_{pL} w'_{pL} + v''_{pL} w_L \quad + w''_{pL} & 0 & 0 \\ 0 & {}'_R \quad v'_{pR} w'_{pR} + v''_{pR} w_R \quad + w''_{pR} & 0 \\ v'_{pL} & v'_{pR} & 1 \end{vmatrix}$$

$$= \frac{{}'_L \quad v'_{pL} w'_{pL} + v''_{pL} w_L \quad + w''_{pL} \qquad {}'_R \quad v'_{pR} w'_{pR} + v''_{pR} w_R \quad + w''_{pR}}{j \hat{D} j} > 0$$

$$\frac{d\, j p_L j}{dC_R} = \frac{1}{\hat{D}} \begin{vmatrix} 0 & 0 & {}'^{0}_L v'_{pL} w_L \\ 0 & {}'_R \quad v'_{pR} w'_{pR} + v''_{pR} w_R \quad + w''_{pR} & {}'^{0}_R v'_{pR} w_R \\ 1 & v'_{pR} & 1 \end{vmatrix}$$

$$= \frac{{}'^{0}_L v'_{pL} w_L \quad {}'_R \quad v'_{pR} w'_{pR} + v''_{pR} w_R \quad + w''_{pR}}{\hat{D}} < 0$$

$$\frac{dp_R}{dC_R} = \frac{1}{\hat{D}} \begin{vmatrix} {}'_L \quad v'_{pL} w'_{pL} + v''_{pL} w_L \quad + w''_{pL} & 0 & {}'^{0}_L v'_{pL} w_L \\ 0 & 0 & {}'^{0}_R v'_{pR} w_R \\ v'_{pL} & 1 & 1 \end{vmatrix}$$

$$= \frac{{}'^{0}_R v'_{pR} w_R \quad {}'_L \quad v'_{pL} w'_{pL} + v''_{pL} w_L \quad + w''_{pL}}{\hat{D}} > 0$$

$$\frac{d}{dC_R} = \frac{1}{\hat{D}} \begin{vmatrix} '_L \left(v'_{pL} w'_{pL} + v''_{pL} w_L + w''_{pL} \right) & 0 & 0 \\ 0 & '_R \left(v'_{pR} w'_{pR} + v''_{pR} w_R + w''_{pR} \right) & 0 \\ v'_{pL} & v'_{pR} & 1 \end{vmatrix}$$

$$= \frac{'_L \left(v'_{pL} w'_{pL} + v''_{pL} w_L + w''_{pL} \right) \; '_R \left(v'_{pR} w'_{pR} + v''_{pR} w_R + w''_{pR} \right)}{|\hat{D}|} < 0$$