News or Noise? The Missing Link

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Abstract

The macroeconomic literature on belief-driven business cycles treats news and noise as distinct representations of people's beliefs about economic fundamentals. We prove that these two representations are actually observationally equivalent. This means that the decision to use one representation or the other must be made on theoretical, and not empirical, grounds. Our result allows us to determine the importance of beliefs as an independent source of fluctuations. Using three prominent models from this literature, we show that existing research has understated the importance of independent shocks to beliefs. This is because representations with anticipated and unanticipated shocks mix the fluctuations due independently to beliefs with the fluctuations due to fundamentals. We also argue that the observational equivalence of news and noise representations implies that structural vector autoregression analysis is equally appropriate for recovering both news and noise shocks.

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1 Introduction

A large literature in macroeconomics has argued that changes in people's beliefs about the future can be an important cause of economic fluctuations. This idea, which dates at least to Pigou (1927), has been more recently mathematically formalized in two different ways. The first way, which we call a "news representation," models people as perfectly observing some (but not all) parts of an exogenous fundamental in advance. By way of analogy, this is like learning for sure today that in next week's big game your favorite team will win the first half. You don't know whether they will win the game, which is ultimately what you care about, because you are still unsure how the second half will turn out. The second way, which we call a "noise representation," models people as imperfectly observing some (possibly all) parts of an exogenous fundamental in advance. This is like your friend telling you that he thinks your team will win next week's game. He follows the sport much more than you do, and is often right, but sometimes he gets it wrong.

At first glance, these two different ways of representing people's beliefs may seem only superficially similar. In both cases, people are getting some advance information about the future. But on a news view they have perfect information and can fully trust whatever information they receive, while on a noise view they have imperfect information and need to solve a signal extraction problem to determine their best forecast. In their recent review of the literature on belief-driven business cycles, Beaudry and Portier (2014) have this to say about the relationship between the two formulations:

> "While these two formulations may appear almost identical, they are actually quite different...To give an idea of the difference, in the [noise]

fact that forward-looking investment decisions often play an important role in the motivation and discussion of belief-driven business cycles.

The observational equivalence of news and noise representations is also relevant to the discussion of whether structural vector autoregression (VAR) analysis is appropriate for recovering noise shocks. We show that in principle, structural VARs can be used to recover noise shocks and their associated impulse responses even though noise representations are not invertible. This is because in both cases, the underlying shocks are only one orthogonal transformation away from the reduced-form representation. An implication of our argument is that invertibility should not be viewed as a necessary condition for the applicability of structural VAR analysis.

We provide one orthogonal transformation that is sufficient to uniquely determine noise shocks (and their associated impulse response functions). This transformation is closely related to a popular thought experiment in the literature on news shocks. The thought experiment is as follows: at date t, agents receive advance information concerning fundamentals at some future date $T > t$. But a surprise innovation at that future date T exactly offsets the advance information agents had previously received. So their expectations end up being incorrect after the fact. This experiment is one way that several authors have tried to separate the effect of beliefs from fundamentals. It turns out that under the set of restrictions we provide, noise shocks generate exactly the combination of offsetting news shocks envisioned by this experiment.

2 Observational Equivalence

News and noise representations are two different ways of describing economic fundamentals and people's beliefs about them. "Fundamentals" are stochastic processes capturing exogenous changes in technology, preferences, endowments, or government policy. Throughout this section, fundamentals are summarized by a single scalar process $\{x_t\}$. People's decisions depend on expected future realizations of x_t, so both representations specify what people can observe at each date and how they use their observations to form beliefs about the future.

The main result of the paper, which is presented in this section, is an observational equivalence theorem relating news and noise representations. To facilitate the exposition, the first subsection presents the result in a simple example with news or noise regarding fundamentals only one period in the future while the second subsection presents the general equivalence result.

2.1 Simple Example

In the simplest of news representations, x_t is equal to the sum of two shocks, $a_{0,t}$ and $a_{1,t-1}$, which are independent and identically distributed (i.i.d.) over time, and which are independent of one another:

$$x_t = a_{0,t} + a_{1,t-1}, \quad \begin{bmatrix} a_{0,t} \\ a_{1,t} \end{bmatrix} \overset{\text{iid}}{\sim} N\left(0, \begin{bmatrix} \sigma^2_{a,0} & 0 \\ 0 & \sigma^2_{a,1} \end{bmatrix}\right). \tag{1}$$

At each date t, people observe the whole history of the two shocks up through that date, $\{a_{0,\tau}, a_{1,\tau}\}$ for all integers $\tau \le t$. Their beliefs regarding fundamentals are rational; the probabilities they assign to future outcomes are exactly those implied

An important feature of our concept of equivalence is that we treat beliefs, as well as fundamentals, as observable. We take this approach for three reasons. First, it is a stronger condition; observational equivalence with respect to a larger set of observables implies observational equivalence with respect to any smaller set of those observables. Second, beliefs are observable in economics, in principle. Beliefs may be measured directly, using surveys, or indirectly, using the mapping between beliefs and actions implied by an economic model. That actions reflect beliefs is, after all, a basic motivation for the literature on belief-driven fluctuations. Third, in a broad class of linear rational expectations models with unique equilibria, endogenous processes are purely a function of current and past fundamentals and beliefs about future fundamentals. So observational equivalence of fundamentals and beliefs implies observational equivalence of the entire economy.

We would also like to emphasize that the observability of beliefs distinguishes our concept of observational equivalence from the typical conception of observational equivalence one often encounters in time series analysis. To use a familiar example (cf. Hamilton, 1994, pp. 64-67) , consider the MA(1) process

$$y_t = \quad _t \qquad _{t\ 1}; \qquad _t \stackrel{\text{iid}}{\ } N(0; \quad ^2);$$

with $j \ j < 1$, and view this as a simple full-information rational expectations model for the determination of the process $f y_t g$ in terms of the exogenous shocks f

Proposition 1. *The news representation of fundamentals and beliefs in system* (1) *is observationally equivalent to the noise representation of fundamentals and beliefs in system* (2) *if and only if:*

$$\sigma_x^2 = \sigma_{a,0}^2 + \sigma_{a,1}^2 \quad \text{and} \quad \frac{\sigma_v^2}{\sigma_x^2} = \frac{\sigma_{a,0}^2}{\sigma_{a,1}^2}.$$

The intuition behind the result comes from the fact that the noise representation implies an observationally equivalent innovations representation (cf. Anderson and Moore, 1979, ch.9) of the form:

$$\begin{aligned} x_t &= \hat{x}_{t-1} + w_{0,t} \\ \hat{x}_t &= \lambda w_{1,t}, \end{aligned} \tag{3}$$

where $\lambda = \sigma_x^2/(\sigma_x^2 + \sigma_v^2)$ is a Kalman gain parameter controlling how much people trust the noisy signal, and $w_t \equiv (w_{0,t}, w_{1,t})'$ is the vector of Wold innovations, which evolves over time according to

$$w_t \overset{\text{iid}}{\sim} N\left(0, \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_x^2 + \sigma_v^2 \end{bmatrix}\right).$$

But system (3) is the same as the news representation in system (1) when $a_{0,t} = w_{0,t}$ and $a_{1,t} = \lambda w_{1,t}$. The news shocks are linear combinations of the Wold innovations.

A direct implication of Proposition (1) is that the news representation is identified if and only if the noise representation is identified. By observational equivalence, both representations have the same likelihood function. Therefore, because the relations in Proposition (1) define a bijection, it is always possible to go from one set of parameters to the other and vice versa.

Corollary 1. *The parameters of the news representation in system* (1) *are uniquely identified if and only if the parameters of the noise representation in system* (2) *are*

2.2 General Equivalence Result

This subsection generalizes the previous example to allow for news and noise at multiple future horizons, and potentially more complex time-series dynamics. To fix notation, we use L^2 to denote the space of (equivalence classes of) random variables with finite second moments, which is a Hilbert space when equipped with the inner product $\langle a, b\rangle = E[ab]$ for any $a, b \in L^2$. Completeness of this space is with respect to the norm $\|a\| \equiv \langle a, a\rangle^{1/2}$. For any collection of random variables in L^2,

$$\{y_{i,t}\}, \quad \text{with } i \in I \subseteq \mathbb{Z} \text{ and } t \in \mathbb{Z},$$

we let $H_t(y)$ denote the closed subspace spanned by the variables $y_{i,\tau}$ for all $i \in I$ and $\tau \in \mathbb{Z}$ such that $\tau \leq t$. To simplify notation, we write $H(y) \equiv H_{\infty}(y)$.

Fundamentals are summarized by a scalar discrete-time process $\{x_t\}$. As in the previous subsection, this process is taken to be mean-zero, stationary, and Gaussian. The fact that fundamentals are summarized by a scalar process is not restrictive; we can imagine a number of different scalar processes, each capturing changes in one particular fundamental. In that case it will be possible to apply the results from this section to each fundamental one at a time.

People's beliefs about fundamentals are summarized by a collection of random variables $\{\hat{x}_{i,t}\}$, with $i, t \in \mathbb{Z}$, where $\hat{x}_{i,t}$ represents the forecast of the fundamental realization x_{t+i} as of time t. Under the assumption of rational expectations, which is maintained throughout this paper, $\hat{x}_{i,t}$ is equal to the mathematical expectation of x_{t+i} with respect to a particular date-t information set. This, together with the fact that fundamentals are Gaussian, implies that the collection $\{\hat{x}_{i,t}\}$ fully characterizes people's entire subjective distribution over realizations of the sequence $\{x_t\}$.

A "representation of fundamentals and beliefs" means a specification of the fundamental process $\{x_t\}$ and the collection of people's conditional expectations about that process at each point in time $\{\hat{x}_{i,t}\}$. A typical assumption is that people's information set is equal to $H_t(x)$, so $\hat{x}_{i,t} \in H_t(x)$ for all $t \in \mathbb{Z}$. In this case, the process $\{x_t\}$ is itself sufficient to describe both the fundamental and people's beliefs about it. A key departure in models of belief-driven fluctuations is that people may have more information than what is reflected in $H(x)$ alone; as a result, $H(x) \subseteq H(\hat{x})$. We will therefore maintain this assumption throughout the paper. We also work exclusively with processes that are regular, in the sense of Rozanov (1967).

Definition 1. In a "news representation" of fundamentals and beliefs, the process $\{x_t\}$ is related to a collection of independent, stationary Gaussian processes $\{a_{i,t}\}$ with $i \in I \subseteq \mathbb{Z}_+$ by the summation

$$x_t = \sum_{i \in I} a_{i,t-i} \quad \text{for all } t \in \mathbb{Z},$$

where people's date-t information set is $H_t(a) \supseteq H_t(x)$.

The idea behind this representation is that people observe parts of the fundamental realization x_t prior to date t. The variable $\varepsilon^a_{i,t} \equiv a_{i,t} - E[a_{i,t}|H_{t-1}(a)]$ is called the "news shock" associated with horizon i whenever $i > 0$. By convention, $0 \in I$, and in that case, the variable $\varepsilon^a_{0,t}$ is referred to as the "surprise shock." An important aspect of this definition is that all of the news shocks are correlated both with fundamentals and people's beliefs. This is because any increase in fundamentals that people observe in advance must generate a one-for-one increase in fundamentals at some point in the future.

Example 1. In the model of Schmitt-Grohe people observe partsjH

The idea behind this representation is that people may receive signals about the fundamental realization x_t prior to date t, but those signals are contaminated with noise. The variable $^{v}_{i;t}$ $v_{i;t}$ $E[v_{i;t}jH_{t\ \ 1}(v)]$ is called the \noise shock" associated with signal i. The variable $^{x}_{t}$ x_t $E[x_t jH_{t\ \ 1}(x)]$ is called the \fundamental shock." An important aspect of this de nition is that all of the noise shocks are completely independent of fundamentals, but because people cannot separately observe $m_{i;t}$ and $v_{i;t}$ at date t, their beliefs are still a ected by noise. The condition that $H_t(s) = H_t(\hat{x})$ simply rules out redundant or totally uninformative signals.

Example 2. In the numerical implementation of their baseline model, Beaudry and Portier (2014) specify the fundamental process $fx_t g$ (in deviations from its mean) and signal process $fs_t g$ as (see their Section 2.1):

$x_t = \ _x x_{t\ \ 1}$ d4 11.9552 Tf 12.]TJ/F11.9ffy 10.9963 2 (s26.652 -1.79

which means that it also provides an explicit computational method for passing from one representation to the other.

The only asymmetric aspect of the theorem involves the uniqueness of the two representations. Any particular news representation will be compatible with several di erent noise representations. This is the same sort of asymmetry present between signal models representations and innovations representations in the literature on state-space models. In general there exist in nitely many signal models with the same innovations representation (cf. Anderson and Moore, 1979, pp. 224-226). We argue in the subsequent sections, however, that despite this multiplicity of noise representations, most interesting economic questions still have a unique answer.

An important implication of Theorem (1) is that associated with any model economy in which fundamentals and beliefs are expressed in the form of a news representation is an observationally equivalent economy in which fundamentals and beliefs are expressed in the form of a noise representation, and vice versa. This is because the observational equivalence of fundamentals and beliefs implies the observational equivalence of any endogenous processes that are functions of fundamentals and beliefs.

in the same model, and as a result, do not properly report the importance of either one. In this section we argue that the observational equivalence result in Theorem (1) is the key to determining the importance of beliefs as an independent cause of fluctuations.

The first subsection explains the problem with using news shocks to determine the importance of beliefs, and the second subsection clarifies the problems that arise when attempting to include both news and noise shocks in the same model. To keep things clear, the discussion of both of these issues is framed in terms of the simple example from Section (2.1). The third subsection establishes an important result regarding the uniqueness of variance decompositions.

3.1 The Problem with News Shocks

In the context of dynamic linear models, the importance of a set of exogenous shocks can be determined by performing a variance decomposition. This entails computing the model-implied variance of an endogenous process under the assumption that all shocks other than those in the set of interest are counterfactually equal to zero almost surely, and comparing that variance to the unconditional variance of the process. More nuanced versions of this exercise include only considering variation over a certain range of spectral frequencies, or variation in forecast errors over a certain forecast horizon. Even in those more nuanced cases, however, the basic intuition is the same.

The problem with using news shocks to determine the importance of beliefs is that news shocks mix changes that are due to fundamentals and changes that are independently due to beliefs. This is because a news shock is an anticipated change in fundamentals. Expectations change at the time the news shock is realized, but then fundamentals change in the future when the anticipated change actually occurs. Of course, people's expectations may not always be fully borne out in the future fundamental, due to other unforeseen disturbances. Nevertheless, the anticipated shock is borne out on average, which is to say that news shocks are related to future fundamentals on average.

A stark way to see this point is to consider the importance of beliefs for driving fundamentals. Because fundamentals are purely exogenous, they are not driven by beliefs at all. However, in the simple news representation from Section (2.1), for example, news shocks can be an arbitrarily large part of fluctuations in the fundamental process $\{x_t\}$. Recall that

$$x_t = a_{0,t} + a_{1,t-1}, \quad \begin{bmatrix} a_{0,t} \\ a_{1,t} \end{bmatrix} \overset{\text{iid}}{\sim} N\left(0, \begin{bmatrix} \sigma^2_{a,0} & 0 \\ 0 & \sigma^2_{a,1} \end{bmatrix}\right).$$

Therefore, the fraction of the variation in $\{x_t\}$ due to news shocks, $\{a_{1,t}\}$ is given by:

$$\frac{\text{var}[x_t | a_{0,t} = 0]}{\text{var}[x_t]} = \frac{\text{var}[a_{1,t}]}{\text{var}[x_t]} = \frac{\sigma^2_{a,1}}{\sigma^2_{a,0} + \sigma^2_{a,1}}.$$

As $\sigma^2_{a,1}$ increases relative to $\sigma^2_{a,0}$, this fraction approaches one, in which case news shocks would explain all the variation in $\{x_t\}$.

To disentangle the importance of beliefs from fundamentals in models with news shocks, we need to use Theorem (1). Specifically, we can write down an observationally equivalent noise representation of the news model, and then use a variance decomposition to compute the share of variation attributable to noise shocks. Because these shocks are unrelated to fundamentals at all horizons, they capture precisely the independent contribution of beliefs.

> \The distinction between pure and realized news is important because one of the promises of the news-driven business cycle literature is to generate \boom-bust" cycles without any observable change in fundamentals ex-post. For understanding whether such \boom-bust" dynamics are quantitatively important it is critical to di erentiate between e ects of news shocks driven by actual news versus movements in endogenous variables caused by realized changes in fundamentals. A traditional variance decomposition does not make this distinction." (p. 2)

The point we would like to make is that the problem is not with the variance decompositions as such; rather, the problem is with the type of shock one considers. It is noise shocks, not news shocks, that are the appropriate shocks for isolating the independent contribution of beliefs. Once that distinction has been made, traditional variance decomposition methods can be employed as usual.

3.2 Mixing News and Noise Shocks

In some cases, researchers have constructed representations of fundamentals and beliefs that seem to include both news and noise shocks at the same time (e.g. Blan-

Therefore, the contribution of the process $\{v_t\}$ is

$$\frac{\mathrm{var}[\hat{x}_t | x_t = 0]}{\mathrm{var}[\hat{x}_t]} = \frac{{}^2_v}{{}^2_x + {}^2_v} = \frac{{}^2\ {}^2}{{}^2 + {}^2} + \frac{{}^2}{{}^2 + {}^2}.$$

The second equality uses the parameteric restrictions from Proposition (3). Because the rst term in this expression is positive, it follows that $\{\ _t\}$ understates the importance of beliefs for explaining variations in $\{\hat{x}_t$

any forecast horizon. This is because the forecast errors are themselves endogenous processes to which Proposition (4) applies.

Corollary 2. *In any noise representation of fundamentals and beliefs, the forecast error variance decomposition of any endogenous process in terms of noise and funda-da-*

Each of the seven exogenous fundamentals is described by the following law of

Applying this proposition to the model of Schmitt-Grohe and Uribe (2012) requires one small step, which is that Proposition (5) is stated for i.i.d. fundamentals, but the fundamentals in system (5) are not i.i.d. However, because $0 < \rho_x < 1$, observing the current and past history of x_t is equivalent to observing the current and past history of the composite disturbance $\varepsilon_t^x \equiv \varepsilon_{0;t}^a + \varepsilon_{4;t-4}^a + \varepsilon_{8;t-8}^a$, which is i.i.d. because each of the news shocks are independent of one another. Therefore, it is possible to treat $\{\varepsilon_t^x\}$ as the fundamental process. By doing so, we arrive at the following corollary.

Corollary 3. *The representation of fundamentals and beliefs in system* (5) *is observationally equivalent to the noise representation*

$$\begin{aligned} x_t &= \rho_x x_{t-1} + \varepsilon_t^x \\ s_{4;t} &= \varepsilon_{t+4}^x + v_{4;t} \\ s_{8;t} &= \varepsilon_{t+8}^x + v_{8;t}, \end{aligned}$$

with the convention that $s_{0;t} \equiv x_t$*, and where*

$$\begin{bmatrix} \varepsilon_t^x \\ v_{4;t} \\ v_{8;t} \end{bmatrix} \overset{iid}{\sim} N\left(0, \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_{v;4}^2 & 0 \\ 0 & 0 & \sigma_{v;8}^2 \end{bmatrix}\right);$$

if and only if

$$\sigma_x^2 = \sigma_{a;0}^2 + \sigma_{a;4}^2 + \sigma_{a;8}^2$$

$$\sigma_5^2$$

Variable	Surprise	News	Fundamental	Noise
Output	57	43	95	5
Consumption	50	50	95	5
Investment	55	45	89	11
Hours	15	85	97	3

Table 1: Variance decomposition (%) in the model of Schmitt-Grohe and Uribe (2012) over business cycle frequencies of 6 to 32 quarters. All variables are in levels. Estimated model parameters are set to their posterior median values.

The main result is that nearly all of the variation in output, consumption, investment, and hours is due to changes in fundamentals. In terms of di erences across the endogenous variables, it is interesting that real investment growth is a ected the least by news shocks, but it is a ected the most by noise shocks. At the same time, hours worked is a ected the most by news shocks and the least by noise shocks. But based on the fact that 90% or more of the variation in every series is attributable to fundamental changes, we conclude that beliefs are not an important independent source of uctuations through the lens of this model.

4.2 Barsky and Sims (2012)

The second model comes from Barsky and Sims (2012). It was constructed to determine whether measures of consumer con dence change in ways that are related to macroeconomic aggregates because of noise (i.e. \animal spirits") or news. The main result of the paper is that changes in consumer con dence are mostly driven by news and not noise. They also nd that that noise shocks account for negligible shares of the variation in forecast errors of consumption and output, while news shocks account for over half of the variation in long-horizon forecast errors. However, as we saw in Section (3.2), including both news and noise shocks in the same model can be misge mhet vi]0 d 0 lid 0-326(to)-326i(s)-1mo theortnrce of beliefs.

The model is a rtndarndoc DSGE") model

setting with time-dependent price rigidity. Fundamentals comprise three different independent processes, which capture exogenous variation in: non-stationary neutral productivity, government spending, and monetary policy. The model is presented in more detail in Appendix (B.2).

People only receive advance information about productivity, and not about the other two fundamentals. So it is only beliefs about productivity that can play an independent role in driving fluctuations. Letting x_t denote the growth rate of productivity (in deviations from its mean), and using our notation from Section (3.2), the process $\{x_t\}$ is assumed to follow a law of motion of the form:

$$\begin{aligned} x_t &= {}_{t-1} + {}_t \\ {}_t &= \; {}_{t-1} + {}_t \\ s_t &= {}_t + {}_t; \end{aligned} \tag{6}$$

where 0 < < 1 and

$$\begin{bmatrix} {}_t \\ {}_t \\ {}_t \end{bmatrix} \overset{iid}{\sim} N\left(0, \begin{bmatrix} {}^2 & 0 & 0 \\ 0 & {}^2 & 0 \\ 0 & 0 & {}^2 \end{bmatrix}\right).$$

Barsky and Sims (2012) refer to $_t$ as a news shock, $_t$ as a surprise shock, and $_t$ as a noise (animal spirits) shock. However, these definitions of news, surprise, and noise shocks are not consistent with the definitions in our paper. To avoid any confusion we will use asterisks to indicate the terminology of Barsky and Sims (2012). So $_t$ is a news* shock, $_t$ is a surprise* shock, and $_t$ is a noise* shock.

The model is estimated by minimizing the distance between impulse responses generated from simulations of the model and those from estimated structural vector autoregressions. The vector autoregressions are estimated on quarterly U.S. data from 1960:Q1-2008:Q4. The time series used to estimate the vector autoregression are: real GDP, real consumption, CPI inflation, a measure of the real interest rate, and a measure of consumer confidence from the Michigan Survey of Consumers (E5Y).

A variance decomposition shows that news* shocks are much more important than noise* shocks. The first column of Table (2) shows the share of business-cycle variation in the level of four endogenous variables that is attributable to surprise* shocks $\{ _t\}$, the second column shows the share attributable to news* shocks $\{ _t\}$, and the third column shows the share attributable to noise* shocks $\{ _t\}$. Due to the presence of government spending and monetary policy shocks, the rows do not sum to 100%; the residual represents the combined contribution of these two additional fundamental shocks. These results are consistent with the authors' original ndings, which are stated in terms of the variance decompositions of forecast errors over di erent horizons, but across all frequency ranges (see their Table 3).

However, to properly isolate the independent contributions of beliefs, it is necessary to construct a noise representation that is observationally equivalent to representation (6). The following proposition presents one such noise representation.

Proposition 6. *The representation of fundamentals and beliefs in system* (6) *is observationally equivalent to the noise representation*

$$x_t = {}_0 m_t + {}_1 m_{t\ 1} + {}_0 m_{t\ 2}$$
$$m_t = {}_1 m_{t\ 1} + {}_2 m_{t\ 2} + {}^{x}_{t}$$
$$s_t = m_t + v_t$$
$$v_t = v_{t\ 1} + {}^{v}_{t} \quad {}^{v}_{t\ 1};$$

with the convention that $s_{0;t}$ x_t*, and where*

$$\begin{bmatrix} {}^{x}_{t} \\ {}^{v}_{t} \end{bmatrix} \overset{iid}{} N \left(0; \begin{bmatrix} {}^{2}_{x} & 0 \\ 0 & {}^{2}_{v} \end{bmatrix} \right);$$

if and only if is equal to the root of the polynomial

$$P(z) = z^2 \quad 1 + {}^2 + \frac{2}{2} \quad z +$$

that lies inside the unit circle, is the root of the polynomial

$$P(z) = z^2 \quad 1 + {}^2 + \frac{{}^2({}^2 + {}^2)}{{}^2\ {}^2} \quad z +$$

that lies inside the unit circle, and

$${}_0 = \frac{2}{2} \qquad {}_1 = {}_0 \frac{1 + {}^2}{}$$
$${}_1 = + \qquad {}_2 =$$
$${}^2_x = \frac{4}{2} \qquad {}^2_v = - {}^2 :$$

Using the noise representation in this proposition, we can re-compute the variance decomposition of the endogenous processes in terms of fundamental shocks and noise consumption uctuations. However, it turns out that what the authors call \noise" shocks do not fully isolate uctuations due to temporary errors in agents' estimates. As a result, it is still an unanswered question what exactly this model implies about the importance of beliefs.

The model is a standard DSGE model with real and nominal frictions: one-period internal habit formation in consumption, investment adjustment costs, variable capacity utilization with respect to capital, and monopolistic price and wage setting with time-dependent price rigidities. Fundamentals comprise six di erent independent processes, which capture exogenous variation in: non-stationary neutral productivity, stationary investment-speci c productivity, government spending, wage markups, nal good price markups, and monetary policy. For more details, see Appendix (B.3).

People only receive advance information about productivity, and not about the other ve fundamentals. So it is only beliefs about productivity that can play an independent role in driving uctuations. Letting x_t denote the growth rate of pro-

indepy uti84.3/(driv4e)-v duev4efromduev4eit(d news shocks. Again, because these definitions are not consistent with the ones in our paper, we will use asterisks to indicate the authors' terminology in contrast to ours.

The model is estimated using likelihood-based methods on a sample of quarterly U.S. data from 1954:Q3-2011:Q1. The time series used for estimation are: real GDP, real consumption, real investment, employment, the federal funds rate, inflation as measured by the implicit GDP deflator, and wages.

A variance decomposition reveals that noise* shocks are important, especially for consumption. The first column of Table (3) shows the share of business-cycle variation in the level of output, consumption, investment, and hours that is attributable to news* shocks, $f \ _t g$ and $f \ _t g$, and the second column shows the share attributable to noise* shocks $f \ _t g$. Due to the presence of the other five fundamental shocks, the rows do not sum to 100%; the residual represents the combined contribution of these additional fundamental shocks. These results are consistent with the authors' original findings, which are stated in terms of the variance decompositions of forecast errors over different horizons (see their Table 6).

However, to properly isolate the independent contribution of beliefs, it is necessary to construct a noise representation that is observationally equivalent to representation (7). The following proposition presents one such noise representation.

Proposition 7. *The representation of fundamentals and beliefs in system* (7) *is observationally equivalent to the noise representation*

$$
\begin{aligned}
x_t &= \quad m_{t+1} + m_t \\
m_t &= \ m_{t \ 1} + \ ^x_t \\
s_t &= \quad (m_t
\end{aligned}
$$

Using the noise representation in this proposition, we can re-compute the variance

In this section, we prove that the observational equivalence of news and noise representations implies that the shocks in any noise representation can be recovered from observables up to an orthogonal transformation. Based on this result, we argue that structural VAR analysis is equally appropriate for recovering the underlying shocks in either a news or noise representation. We also explore one particular orthogonalization that is related to the popular "news-reversal" thought experiment that the existing literature has used to describe boom-bust episodes in models with news shocks.

5.1 Recovering Shocks and Invertibility

When can the underlying shocks in news and noise representations be recovered from the data? That is, given data on fundamentals and beliefs, when are the underlying shocks in these representations uniquely determined? The uniqueness of the news representation according to Theorem (1) implies that in any news representations of fundamentals and beliefs, each underlying shock is uniquely determined.

By contrast, each underlying shock in a noise representation is not uniquely determined. Proposition (4) and its associated Corollary (2) establish the uniqueness of variance decompositions computed in terms of fundamentals and noise, but they do not imply that it is possible to separately recover each individual shock. However, we can prove the following result, which says that the shocks are determined up to an orthogonal transformation.

Proposition 8. *In any noise representation of fundamentals and beliefs, the space spanned by the underlying shocks at each date is uniquely determined.*

An important concept in discussions regarding the applicability of structural VAR analysis is that of invertibility. This has to do with whether or not it is possible to express one collection of stochastic processes as a linear combination of the current and past history of another collection of stochastic processes.

Definition 4. A collection of stochastic processes $\{y_{i,t}\}$, $i \in I_y \subseteq \mathbb{Z}_+$ is "invertible" with respect to the collection of shock processes $\{\varepsilon_{i,t}\}$, $i \in I_\varepsilon \subseteq \mathbb{Z}_+$, if

$$H_t(\varepsilon) = H_t(y) \quad \text{for all } t \in \mathbb{Z}.$$

Based on this definition, we refer to a representation of the collection of $\{y_{i,t}\}$ as an "invertible representation" if $\{y_{i,t}\}$ is invertible with respect to all the underlying shocks in that representation.[4] Also, note that we use the term "shock process" to refer to a process that is uncorrelated over time.

Our second theoretical result of this subsection characterizes news and noise representations in terms of invertibility.

Proposition 9. *Any news representation of fundamentals and beliefs is invertible, but any noise representation is not invertible.*

This result is a generalization of the one that Blanchard, L'Huillier, and Lorenzoni (2013) prove in the context of a simple model of consumption determination, and the basic intuition is the same. If noise representations were invertible, then people would be able to distinguish the informative parts of their signals from the noise. By rationality, noise shocks could never affect people's beliefs. But then it would not be possible to recover those shocks from the current and past history of people's beliefs.

Any collection of observable processes is invertible with respect to infinitely many different collections of underlying shocks. However, these shocks have the important property that they are all related by an orthogonal transformation. We state this in the following proposition, which has a well-known finite-dimensional counterpart (e.g. Rozanov, 1967, p. 57):

Proposition 10. *If a collection of stochastic processes is invertible with respect to two different collections of shock processes, then the space spanned by those shocks at each date is the same.*

5.2 Using Structural VAR Analysis

For many researchers, Proposition (9) settles the question of whether structural VAR analysis can be used to recover shocks in news and noise representations. For example, the central methodological argument of Blanchard, L'Huillier, and Lorenzoni (2013) is that structural VAR analysis is not applicable for recovering noise shocks due to non-invertibility:

[4] What we call invertibility is sometimes called "fundamentalness" (cf. Rozanov, 1967, ch. 2).

> “[In situations with] a partially informative signal, the reduced-form VAR representation is non-invertible and a structural VAR approach cannot be used.” (p. 3051)

However, in this subsection we argue that the applicability of structural VAR analysis should not be understood solely in terms of invertibility.

Structural VAR analysis has two steps: a VAR step and a structural step. The VAR step defines a “reduced-form” representation of the observables with residuals that come from a projection of observables on their past history. The structural step uniquely determines a collection of “economic shocks” from the reduced-form representation by using theoretical restrictions to pin down a single orthogonal transformation (an orthogonal matrix in the finite-dimensional case). In other words, we can say that structural VAR analysis is applicable whenever knowledge of the reduced-form representation is at most one orthogonal transformation away from knowledge of the economic shocks.

To be more precise, we can define a reduced-form representation of fundamentals and beliefs in the following way.

Definition 5. In a “reduced-form” representation of fundamentals and beliefs,

$$\hat{x}_{i,t} = \check{x}_{i,t-1} + \tilde{\ }_{i,t} \quad \text{for all } i,t \in \mathbb{Z},$$

where $\check{x}_{i,t-1} \in H_{t-1}(\hat{x})$ and $\tilde{\ }_{i,t} \perp H_{t-1}(\hat{x})$.

The first step of structural VAR analysis is to treat this representation as known. The second step is to determine whether the reduced-form shocks $\{\tilde{\ }_{i,t}\}$ uniquely determine the space spanned by the underlying shocks in either a news or noise representation.

For a news representation, the answer is yes. Note that, by construction, $\{\hat{x}_{i,t}\}$ is invertible with respect to $\{\tilde{\ }_{i,t}\}$. By Propositions (9) and (10), it follows that the space spanned by the reduced-form shocks is equal to the space spanned by the shocks in any news representation are the same at each date. Therefore, the reduced-form shocks uniquely determine the space spanned by the shocks in any news representation.

Interestingly, for a noise representation, the answer is also yes. To see why, note that Proposition (8) implies that the space spanned by the shocks in any noise representation is uniquely determined by $\{\hat{x}_{i,t}\}$. But then that space must also be also uniquely determined by the reduced-form residuals, because $H_t(\tilde{\ }) = H_t(\hat{x})$ for all $t \in \mathbb{Z}$ by invertibility.

But isn't it true that in a news representation, the shocks themselves are uniquely determined (by Theorem (1)), while in a noise representation only the space spanned by the shocks is uniquely determined (by Theorem (1) and Proposition (8))? Yes, but that is only because the orthogonal transformation linking the reduced-form representation to the news representation is already embedded in the definition of a news representation. Of course, we could have just as easily appended one particular orthogonal transformation to the definition of a noise representation in the first place. Therefore, we can conclude that to recover the underlying shocks in both news and noise representations from the reduced-form representation, the same theoretical input is required: one orthogonal transformation.

One natural set of restrictions is that noise shocks are orthogonal, and that noise shock $i \in I$ has a unit impact response on the forecast $\hat{x}_{i,t}$ but zero impact response on forecasts $\hat{x}_{j,t}$ for $j < i$. These restrictions impose a familiar lower-triangular structure on the shocks in a noise representation. They amount to a recursive causal ordering of the noise shocks in terms of the observable collection of forecasts $\{\hat{x}_{i,t}\}$.

Assumption 1. *In any noise representation of fundamentals and beliefs, the following conditions are satisfied:*

(a) ${}^{v}_{i,t} \perp {}^{v}_{j,t}$ *for all* $i \neq j \in I$,

(b) $\langle \hat{x}_{i,t}, {}^{v}_{i,t} \rangle / \| {}^{v}_{i,t} \|^2 = 1$ *for all* $i \in I$, *and*

(c) $\langle \hat{x}_{j,t}, {}^{v}_{i,t} \rangle = 0$ *for all* $j < i \in I$.

Proposition 11. *In any noise representation of fundamentals and beliefs that satisfies Assumption* (1)*, the underlying shocks are uniquely determined.*

Of course, an immediate corollary of this proposition is that under Assumption (1), the impulse response function and variance decomposition of any endogenous process with respect to each shock in a noise representation are also uniquely determined.

5.3 Offsetting News Shocks

The orthogonal transformation implicit in Assumption (1) is also related to a popular thought experiment in the news-shock literature, which some researchers have used to try to isolate the effects of a change in beliefs that does not correspond to any change in fundamentals (e.g. Christiano et al. (2010) Section 4.2, Schmitt-Grohe and Uribe (2012) Section 4.2, Barsky, Basu, and Lee (2015) Section IV.A, or Sims (2016) Section 3.3). This experiment involves computing the impulse responses of endogenous variables in response to particular combinations of offsetting news and surprise shocks. The following description comes from Christiano et al. (2010), with slight modification to match the notation in this paper:

> "In the first period, a signal, $^{a}_{n;t} > 0$ arrives, which creates the expectation that z_t n periods later will jump. However, that expectation is ultimately disappointed, because $^{a}_{0;t} = \ ^{a}_{n;t}$. Thus, in fact nothing real ever happens. The dynamics of the economy are completely driven by an optimistic expectation about future fundamentals, an expectation that is never realized." (p. 116)

It turns out that in models with i.i.d. news shocks, the noise shocks in any observationally equivalent representation generate exactly the sort of offsetting news shocks envisioned by this thought experiment. To see this, recall the simple news representation in system (1) from Section (2.1). The noise representation in system (2) does not satisfy part (b) of Assumption (1), because $\hat{x}_{1;t} = \ s_t$, so $h\hat{x}_{1;t}; v_t i = k$

While this discussion shows that it may be possible to find particular linear combinations of news shocks that mimic a noise shock, there are a number of advantages to working directly with noise shocks. First, we can think about how likely these situations arise, since we have an explicit probability distribution for the noise shocks: for example, how big is a "one standard deviation impulse?" Second, we can ask how important these types of situations are in the data sample overall; that is, we can do a proper variance decomposition. Third, the fact that noise representations are generally not unique helps us to remember that the dynamic response of the economy to noise shocks is also not unique. With news shocks at multiple different horizons, there are many ways people's expectations can be subsequently reversed, and the reversal may not occur only in the final period.

6 Conclusion

Models with news and noise are more intimately related than the literature has acknowledged. In fact, as we have argued here, there is a precise sense in which they are identical. The missing link is the observation that they are really just two different ways of describing the joint dynamics of exogenous economic fundamentals and people's beliefs about them. This observation is formalized by Theorem (1).

Far from being a purely negative result, the observational equivalence between news and noise representations serves an important positive purpose. Namely, it provides a way to determine the importance of beliefs as an independent cause of fluctuations. A number of prominent studies have constructed models to understand how beliefs can drive fluctuations. However, none of them has fully isolated the contribution of beliefs that is independent of the contribution due to fundamentals. This is because what these studies refer to as "news" shocks actually mix the fluctuations due fundamentals and beliefs.

In order to disentangle beliefs from fundamentals, it is necessary to first derive a noise representation of the model, and then perform variance decompositions in terms of noise shocks. These decompositions are always unique by Proposition (4). We also state a set of sufficient conditions for uniquely recovering the impulse response function of any endogenous process with respect to noise shocks. The uniqueness result that obtains under those conditions is presented in Proposition (11).

We then apply our results to three quantitative models of the U.S. economy, from

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A Proofs

Proof of Proposition (1). Let $\hat{x}_t \equiv E_t[x_{t+1}]$ denote people's expectations of the fundamental at date $t+1$ given their information up through date t. The observable processes are $\{x_t\}$ and $\{\hat{x}_t\}$. Expectations at horizons greater than one are spanned by these two processes.

The two representations are observationally equivalent if and only if the covariance generating function (c.g.f.) of the data vector $d_t \equiv (x_t, \hat{x}_t)'$ is the same under either representation. Let $g_d(z)$ denote the c.g.f. of d_t, where z is a number in the complex plane. Then we can equate the c.g.f.'s implied by each representation:

$$g_d(z) = \underbrace{\begin{bmatrix} \sigma^2_{a,0} + \sigma^2_{a,1} & \sigma^2_{a,1} z \\ \sigma^2_{a,1} z^{-1} & \sigma^2_{a,1} \end{bmatrix}}_{\text{news}} = \begin{bmatrix} \sigma^2_x & \frac{\sigma^4_x}{\sigma^2_x + \sigma^2_v} z \\ \frac{\sigma^4_x}{\sigma^2_x + \sigma^2_v} z^{-1} & \end{bmatrix}$$

orthogonal sequence of shocks (cf. Luenberger, 1969, Theorem 3.5.1). Specifically, let us define:

$$
\begin{aligned}
{}^{a}_{0;t} &= w_{0;t} \\
{}^{a}_{i;t} &= w_{i;t} \quad \sum_{j=0}^{i-1} {}_{i;j} \, {}^{a}_{j;t} \quad \text{for } i > 0;
\end{aligned}
$$

where ${}_{i;j}$ $\langle w_{i;t}; {}^{a}_{j;t} \rangle / \| {}^{a}_{j;t} \|^2$ is a projection coefficient. Define the index set I to be the set of indices $i \in \mathbb{Z}_+$ such that $\| {}^{a}_{i;t} \| > 0$. The collection

Because $H(x) \subseteq H(a)$, there exist unique elements $m_{i,t} \in H(x)$ and $v_{i,t} \in H(s) \ominus H(x)$ such that

$$s_{i,t} = m_{i,t} + v_{i,t}.$$

This defines a noise representation when people's date-t information set is $H_t(s)$. What remains is to prove that the expectations implied by this noise representation are the same as the ones implied by the original news representation. Because $H_t(s) = H_t(a)$ by construction, and $H_t(a) =$

To consider variance decompositions at different frequencies, let $f_y(\omega)$ denote the spectral density function of a stochastic process $\{y_t\}$. Then because $a_t \perp b_t$ for all $t \in \mathbb{Z}$, it follows that

$$f_c(\omega) = f_a(\omega) + f_b(\omega);$$

where the functions $f_a(\omega)$ and $f_b(\omega)$ are uniquely determined by the processes $\{a_t\}$ and $\{b_t\}$. These functions in turn uniquely determine the share of the variance of $\{c_t\}$ due to noise shocks in any frequency range $\underline{\omega} < \omega < \bar{\omega}$, which is equal to

$$\frac{\int_{\underline{\omega}}^{\bar{\omega}} f_b(\omega)d\omega}{\int_{\underline{\omega}}^{\bar{\omega}} f_c(\omega)d\omega}.$$

The share due to fundamentals is equal to one minus this expression. □

Proof of Corollary (2). Consider an arbitrary noise representation of fundamentals and beliefs, and an endogenous process $\{c_t\}$. By the rationality of expectations, people's best forecast of

Equating the c.g.f.'s in (11) with those in (12), and recursively solving for the parameters of the noise representation delivers the relations stated in the proposition. □

Proof of Corollary (3). Define the composite shock

$$\varepsilon_t^x \equiv \varepsilon_{0,t}^a + \varepsilon_{4,t-4}^a + \varepsilon_{8,t-8}^a . \tag{13}$$

The process $\{\varepsilon_t^x\}$ is i.i.d. because $\{\varepsilon_{i,t}^a\}$ is i.i.d. for each $i \in I \equiv \{0, 4, 8\}$. People's date-$t$ information set in representation (5) is $H_t(\varepsilon^a)$. But based on this information set, equation (13) defines a news representation for $\{\varepsilon_t^x\}$

which is in canonical form (cf. Whittle, 1983, ch. 2). This means that fm_tg has an ARMA(2,0) representation of the form:

$$m_t = \;_1 m_{t\;\;1} + \;_2 m_{t\;\;2} + \;^x_t; \quad \;^x_t \;^{\text{iid}}\; N(0; \;^2_x);$$

where $_1$ $+$, $_2$, and ^2_x ^{4}=(^{2}). Inverting the relation in Equation (14), we have that $x_t = \;(L)\;^1 m_t$. Expanding the lag polynomial, it follows that fx_tg has a representation of the form:

$$x_t =$$

Using the nite-order ARMA restrictions in system (7), it follows that $\bar{m}_t$ is related to $f x_t g$ according to:

$$\bar{m}_t = \quad (L)x_t; \qquad (z) \qquad\qquad (1 \quad z^{1})$$

these two representations are observationally equivalent. Uniqueness follows from the uniqueness of the processes $\{m_t\}$ and $\{v_t\}$ in terms of $\{\tilde{m}_t\}$ and $\{\tilde{v}_t\}$ in the orthogonal projection of $s_{1;t}$ on $H(x)$ at each date, and the fact that the polynomial de ning $_1$ and $_2$ only has two roots inside the unit circle. □

Now consider an arbitrary noise representation. Suppose that it is invertible. Then $^{v}_{i,t} \in H_t(\hat{x})$ for any $i \in I \quad \mathbb{Z}_+$ and $t \in \mathbb{Z}$. This implies that the noise shocks are contained in the information set of agents. But by rationality, if $^{v}_{i,t}$ is contained in the information set of agents at date t, because it is uncorrelated with fundamentals, $^{v}_{i,t} \notin H_t(\hat{x})$. This is a contradiction. Therefore, the representation is not invertible. □

Proof of Proposition (10). By invertibility, $H_t(\ ) = H_t(y) = H_t(\tilde{\ })$ for all $t \in \mathbb{Z}$. Then by the uniqueness of orthogonal projections:

$$H_t(\ ) \quad H_{t-1}(\ ) = H_t(\tilde{\ }) \quad H_{t-1}(\tilde{\ }) \quad \text{for all } t \in \mathbb{Z}.$$

□

Proof of Proposition (11). By definition, the collection of noise shocks $\{ ^{v}_{i,t}\}$ for $i \in I$ and fixed $t \in \mathbb{Z}$ generates the subspace $D_t \quad H_t(v) \quad H_{t-1}(v)$. By Proposition (8), $H_t(v) = H_t(\hat{v})$, so $D_t = H_t(\hat{v}) \quad H_{t-1}(\hat{v})$. Therefore, the shock $\hat{\ }^{v}_{i,t} \quad \hat{v}_{i,t} \quad E[\hat{v}_{i,t}|H_{t-1}(\hat{v})]$ can be represented in the form:

$$\hat{\ }^{v}_{i,t} = \sum_{j \in I} \ _{i,j} \ ^{v}_{j,t}, \tag{15}$$

where $_{i,j} \quad \langle \hat{\ }^{v}_{i,t}, \ ^{v}_{j,t} \rangle / \| \ ^{v}_{j,t} \|^2$. By Assumption (1), the collection $\{ ^{v}_{i,t}\}$ for fixed t is an orthogonal basis for D_t with $_{i,i} = 1$ and $_{i,j} = 0$ for all $i < j \in$

B Quantitative Models

The following subsections provide a sketch of each of the three quantitative models considered in this paper. For more details, we refer the reader to the original articles and their supplementary material.

B.1 Model of Schmitt-Grohe and Uribe (2012)

A representative household chooses consumption $\{C_t\}$, labor supply $\{h_t\}$, investment $\{I_t\}$, and the utilization rate of existing capital $\{u_t\}$ to maximizes its lifetime utility subject to a standard budget constraint:

$$\max \quad E \sum_{t}^{\infty}$$

B.2 Model of Barsky and Sims (2012)

A representative household chooses consumption $\{C_t\}$, labor supply $\{N_t\}$, and real holdings of riskless one-period bonds $\{B_t\}$ to maximize its lifetime utility subject to a standard budget constraint:

$$\max \quad E \sum_{t=0}^{\infty} {}^{t} \left[\ln(C_t - C_{t-1}) - \frac{N_t^{1+1/}}{1+1/} \right] \quad \text{subject to}$$

$$C_t + B_t = w_B^B$$

B.3 Model of Blanchard, L'Huillier, and Lorenzoni (2013)

A representative household chooses consumption $\{C_t\}$, investment $\{I_t\}$, nominally risk-free bond holdings $\{B_t\}$, and the rate of capital utilization $\{U_t\}$ to maximize its lifetime utility subject to a standard budget constraint:

$$\max \; E \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - hC_{t-1}) - \frac{1}{1+\zeta} \int_0^1 N_{j,t}^{1+\zeta} \, dj \right] \quad \text{subject to}$$

$$P_tC_t + P_tI_t + T_t + P_tC(U_t)K_{t-1} = R_{t-1}B_{t-1} + \Pi_t + \int_0^1 W_{jt}N_{jt}dj + R_t^kU_tK_{t-1},$$

$$K_t = (1-\delta)K_{t-1} + D_t[1 - G(I_t/I_{t-1})]I_t$$

where P_t is the price level, T_t is a lump sum tax, R_t is the gross nominally risk-free rate, Π_t is aggregate profits, W_{jt} is the wage of labor type j, R_t^k is the capital rental rate, $0 < \delta < 1$ is the rate of depreciation, $G(I_t/I_{t-1})$ represents investment adjustment costs, $C(U_t)$ represents the marginal cost of increasing capacity utilization. It also chooses the wage $\{W_{jt}\}$ for each type of labor subject to the constraint that it will only be able to re-optimize its wage each period with constant probability $1-\theta_w$.

Final goods producers are competitive and take the price of intermediate goods as given, P_{jt}, and each have a production function of the form

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{1+\mu_{pt}}} dj \right]^{1+\mu_{pt}}.$$

Intermediate goods firms are monopolistically competitive, each with a production function of the form $Y_{jt} = (K_{jt})^{\alpha}(A_tL_{jt})^{1-\alpha}$. Each intermediate firm chooses a price for its own good, subject to a $1-\theta_p$ probability of re-optimization each period.

Labor services are supplied to intermediate goods producers by competitive labor agencies that take wages as given, W_{jt}, and have a production function of the form

$$N_t = \left[\int_0^1 N_{jt}^{\frac{1}{1+\mu_{wt}}} dj \right]^{1+\mu_{wt}}.$$

Market clearing in the final goods market requires that $C_t + I_t + C(U_t)K_{t-1} + G_t = Y_t$, and in the labor market that $\int_0^1 L_{jt}dj = N_t$. Monetary policy follows the rule:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\psi_\pi \pi_t + \psi_y \hat{y}_t) + q_t.$$

The six fundamental processes capture exogenous variation in permanent neutral productivity $\{A_t\}$, transitory investment-specific productivity $\{D_t\}$, price markups $\{\mu_{pt}\}$, wage markups $\{\mu_{wt}\}$, government spending $\{G_t\}$, and monetary policy $\{q_t\}$.